Master in Economics
Lecture 1: Small Open Economies

International Business Cycle

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Content of the class

The class will consist of 3 separate sections:

1. International Business Cycles (Lopez)
2. Exchange Rates and International Portfolios (Michalski)
3. Sovereign Debt (Mengus)

International Business Cycles

For the first part of the class we will cover 4 main topics

1. Small Open Economies
2. Two-country models
3. International risk sharing
4. Models with heterogeneous agents

Key Questions

- What are the features and causes of business cycles in emerging economies (are they different from developed economies?)
- Can standard business cycles models explain international transmission of shocks and cycles in regions/set of countries?
- Do countries share risk? How effectively?
- What are the propagation mechanisms of international shocks? Do micro-structure matters?

Small Open Economies

- Small Open Economies (SOE) are countries for which international trade/ finance are an important part of the economy and international prices are taken as given (terms of trade, interest rates, oil prices) .

- SOE also is used to describe models that are not general, but partial equilibrium as one variable (e.g. interest rates) is exogenous to the system

- When comparing stylized features of SOE business cycles there is a pattern that emerges between high and middle/low income countries (e.g. Canada vs Argentina).

- The labels of "developed" and "emerging" drawn from per-capita income levels are very often used (S&P and IFC classification) but have been recently criticized (e.g. Portugal vs Chile or South Korea vs Spain) [link FT Article]

Small Open Economies Business Cycles

	Emerging Economies	Developed Markets
$\sigma(Y)$	2.74	1.34
$\sigma(\triangle Y)$	1.87	.95
$\rho(Y)$	.76	.75
$\sigma(C)/\sigma(Y)$	1.45	.94
$\sigma(I)/\sigma(Y)$	3.91	3.41
$\rho(TB/Y, Y)$	-51	-.17
$\rho(C, Y)$	.72	.66

Notes: Averages for 13 emerging and 13 developed economies. Quarterly data detrended using Hodrick-Prescott filter. The emerging economies are: Argentina, Brazil, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, Turkey. Developed countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zeleand, Norway, Portugal, Spain, Sweden, Switzerland. Source: Aguiar and Gopinath (2007)

Standard SOE

- The starting point of the standard SOE is an endowment economy (see [Notes Schmitt-Grohe Uribe])
- Infitinitely-lived representative household that maximizes consumption (c) subject to stochastic exogenous endowment (y) and with access to international capital markets in the form of non-contigent (risk-free) real debt (d) that pays a constant interest rate (r).

$$max_{\{c_t,d_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{1}$$

$$d_t = (1+r)\, d_{t-1} + c_t - y_t \tag{2}$$

$$lim_{j\to\infty} \frac{d_{t+j}}{(1+r)^j} \leq 0 \tag{3}$$

Consumption Dynamics

$$U(c_t) = \beta (1+r) E_t U(c_{t+1}) \tag{4}$$

- We need to assume $\beta(1+r) = 1$ otherwise consumption will drift permanently (long-rung growth or decay)
- Let's assume quadratic preferences:

$$U(c_t) = -\frac{1}{2}(c_t - \bar{c})^2 \tag{5}$$

- The Euler equation becomes:

$$c_t = E_t c_{t+1} \tag{6}$$

Intertemporal Resource Constraint

- Iterating forward the budget constraint at time t, one can find the intertemporal resource constraint of the economy

$$(1+r)\,d_{t-1} = \frac{d_{t+j}}{(1+r)^j} + \sum_{s=0}^{j} \frac{y_{t+s} - c_{t+s}}{(1+r)^s} \tag{7}$$

- Applying conditional expectations time t, taking the limit $j \to \infty$ and using the transversality condition

$$(1+r)\,d_{t-1} = E_t \sum_{s=0}^{\infty} \frac{y_{t+s} - c_{t+s}}{(1+r)^s} \tag{8}$$

- The country's initial net foreign debt position equals the expected discount of current and future savings.

$$rd_{t-1} + c_t = \frac{r}{1+r} E_t \sum_{s=0}^{\infty} \frac{y_{t+s}}{(1+r)^s} \tag{9}$$

Equilibrium Allocation

- Following the literature, let's assume the endowment is an AR(1) process:

$$y_t = \rho y_{t-1} + \epsilon_t \tag{10}$$

- With $\rho < 1$, ϵ_t i.d.d. innovations. We solve for consumption:

$$c_t = \frac{r}{1 + r - \rho} y_t - r d_{t-1} \tag{11}$$

- The Trade Balance $(tb_t = y_t - c_t)$

$$tb_t = \frac{1 - \rho}{1 + r - \rho} y_t + r d_{t-1} \tag{12}$$

CA and Debt Dynamics

- Current Account ($ca_t \equiv -(d_t - d_{t-1}) = -rd_{t-1} + tb_t$)

$$ca_t = \frac{1-\rho}{1+r-\rho} y_t \tag{13}$$

- The evolution of the debt is

$$d_t = d_{t-1} - \frac{1-\rho}{1+r-\rho} y_t \tag{14}$$

Main features SOE

- If shocks are not perfectly persistent ($\rho < 1$) :
  - Consumption responds less than output
  - Trade Balance and CA improve upon positive shock (pro-cyclical)
  - Debt falls with a positive shock

- If shocks are perfectly persistent ($\rho \to 1$) :
  - Consumption increases one-to-one
  - Trade Balance and CA remain unchanged
  - Debt remains constant

- The endogenous variables of the model are Random Walks.

- There are 3 ways to induce stationarity often used in the literature (see Schmitt-Grohé and Uribe (2003))

Solving the Stationarity Problem

- Endogenous Discount Factor: The discount factor (β) is not constant but a function of consumption: $\beta(c_t)$ with $\beta_c < 0$. In steady-state $\beta(c)(1+r) = 1$, which pins down the steady-state level of consumption as function of r and the parameters defining $\beta(c)$
  - e.g: $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\beta(c) = [1+c]^{-\psi_1}$ as in Mendoza (1991)

- Debt Elastic Interest Rate Premium: the interest rate is increasing in the net foreign debt position: $\beta(1+r_t(d_t)) = 1$. This helps to select the level of debt in steady-state.
  - e.g: $r_t = r + p(d_t)$ with $p(d_t) = \psi_2\left(e^{d_t-\bar{d}} - 1\right)$.

Solving the Stationarity Problem II

- Convex portfolio adjustment costs:

$$d_t = (1 + r)\, d_{t-1} + c_t - y_t + \frac{\psi_3}{2}\left(d_t - \bar{d}\right)^2 .$$

- The Euler equation becomes:

$$U(c_t)\left[1 - \psi_3\left(d_t - \bar{d}\right)\right] = \beta (1 + r)\, U(c_{t+1})$$

- In steady-state:

$$\beta (1 + r) = 1$$

$$d_t = \bar{d}$$

Adding Capital to the model

- In the standard SOE the only way to smooth shocks over time is by adjusting the trade balance.
- In reality, households have access to domestic technologies to save.

Non-stochastic Model with Capital

$$max_{\{c_t, i_t, b_t\}_{t=0}^{\infty}} \sum \beta^t U(c_t) \tag{15}$$

$$b_t = (1+r)\, b_{t-1} + y_t - c_t - i_t \tag{16}$$

$$y_t = A_t F(k_t) \tag{17}$$

$$k_{t+1} = k_t + i_t \tag{18}$$

$$lim_{j \to \infty} \frac{b_{t+j}}{(1+r)^j} \geq 0 \tag{19}$$

FOC

$$\lambda_t = \beta (1 + r) \lambda_{t+1} \tag{20}$$

$$\lambda_t = \beta \lambda_{t+1} \left[A_{t+1} F_k' (k_{t+1}) + 1 \right] \tag{21}$$

$$b_t = (1 + r) b_{t-1} + A_t F (k_t) - c_t - k_{t+1} + k_t \tag{22}$$

With the assumption $\beta (1 + r) = 1$

$$c_t = c_{t+1} \tag{23}$$

The two equilibrium conditions:

$$r = A_{t+1} F_k' (k_{t+1}) \tag{24}$$

$$c_t = r b_{t-1} + \frac{r}{1 + r} \sum_{s=0}^{\infty} \frac{A_{t+s} F_k' (k_{t+s}) - k_{t+s+1} - k_s}{(1 + r)^s} \tag{25}$$

Steady-state

Because there is no depreciation, investment in steady-state is zero. All the variable in the model are determined by $\bar{A}$ and r.

$$c_{ss} = \bar{c} \qquad i_{ss} = 0 \qquad y_{ss} = \bar{A}F\left(\bar{k}\right)$$
$$b_{ss} = \frac{\bar{c}-\bar{y}}{r} \qquad k_{ss} = f\left(\bar{A}, r\right) \qquad ca_{ss} = 0 \tag{26}$$
$$tb_{ss} = -r\bar{b}$$

Permanent Productivity Shocks

- Consider $\bar{A}' > \bar{A}$ in period 0 and thereafter.
- Consumption increases permanently. Capital stock should increase but it is fixed at time 0. Investment jumps so the economy reaches the new k'.
- The trade balance deteriorates at time 0 and improves at time 1.

$$c_0 = r\bar{b} + \frac{r}{1+r}\left[A'F\left(\bar{k}\right) + \left(k' - \bar{k}\right)\right] + \frac{1}{1+r}A'F\left(K'\right)$$

$$tb_0 = -r\bar{b} - \frac{1}{1+r}\left[A'F\left(k'\right) - A'F\left(\bar{k}\right) + \left(k' - \bar{k}\right)\right]$$

Transitory Productivity Shocks

- Consider $\bar{A}' > \bar{A}$ in period 0 and thereafter goes back to $\bar{A}$.
- Consumption increases temporarily but the capital stock remains unchanged (and so investment). The extra-output is used to invest in international markets . The trade balance improves at time 0 and deteriorates thereafter.

$$c_0 = c_{-1} + \frac{r}{1+r}\left[A' - \bar{A}\right] F\left(\bar{k}\right) \tag{27}$$

$$tb_0 - tb_{-1} = \frac{1}{1+r}\left[\left(A' - \bar{A}\right) F\left(\bar{k}\right)\right] \tag{28}$$

- The more persistent the productivity shocks, the more likely the trade balance will experience a deterioration at the time of a positive shock.
  - Adjustment costs of capital could offset this initial effect because investment will increase slowly
  - How much persistence and how large adjustment costs? This is a quantitative question

Real Business Cycle Model (Mendoza 1991)

- Let's consider the standard RBC model as in Mendoza (1991)

$$max_{\{c_t, i_t, b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t) \tag{29}$$

$$d_t = (1 + r_t) d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) \tag{30}$$

$$ln(A_{t+1}) = \rho ln(A_t) + \epsilon_{t+1} \tag{31}$$

with the following functional forms:

$$U(c,h) = \frac{\left[c - \frac{h^\omega}{\omega}\right]^{1-\gamma} - 1}{1-\gamma} \qquad r_t = r + \psi_2\left(e^{d_t - \bar{d}} - 1\right)$$

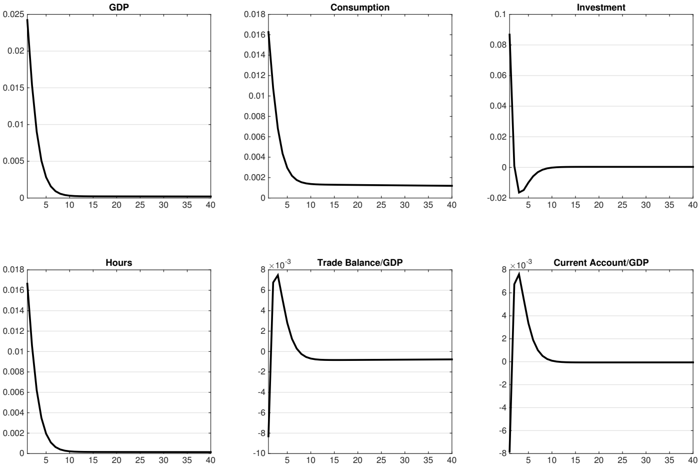
$$y = Ak^\alpha h^{1-\alpha} \qquad \Phi(k_{t+1} - k_t) = \phi\frac{(k_{t+1} - k_t)^2}{2}$$

Calibration RBC

Parameter	Value	Description
γ	2	Inter-temporal elasticity of substitution
ω	1.45	Labor Supply Elasticity
ψ_2	.000742	Discount Factor Parameter
α	.32	Capital Share
ϕ	0.028	Adjustment Cost
β	0.96	Real Interest Rate
δ	.1	Depreciation Rate
ρ	.42	Persistence TFP shock
σ_ϵ	0.0129	Volatility Innovations TFP
$\bar{d}$	.7442	Debt

Notes: Source: Mendoza (1991)

Impulse Response Functions



Small Open Economies Business Cycles

	Canada			Model		
	Volatility	σ_x/σ_y	$\rho_{x,y}$	Volatility	σ_x/σ_y	$\rho_{x,y}$
y	2.8	1	1	3.08	1	1
c	2.5	0.89	0.59	2.71	0.88	0.84
i	9.8	3.50	0.64	9.04	2.94	0.66
h	2	0.71	0.8	2.12	0.69	1
$\frac{tb}{y}$	1.9	0.68	-0.13	1.78	0.58	-0.04

Notes: Own calculations and Mendoza (1991)

Role of Persistence and Adjustment Costs

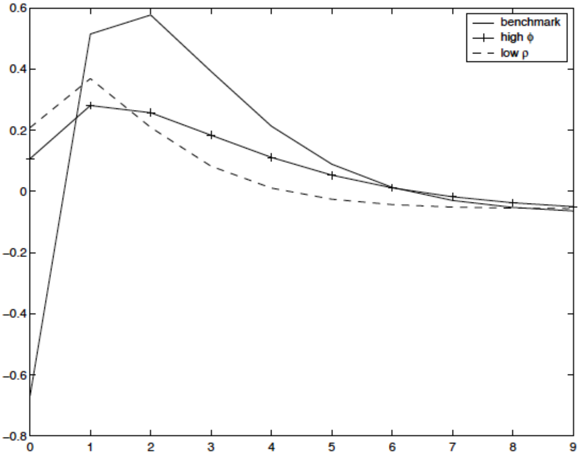


Figure : Impulse Response Function TB/Y (productivity shock)

The Cycle is the Trend (Aguiar-Gopinath 2007)

- Shocks to the trend of productivity could be the primary source of fluctuations
- This requires to distinguish between transitory and permanent shocks.

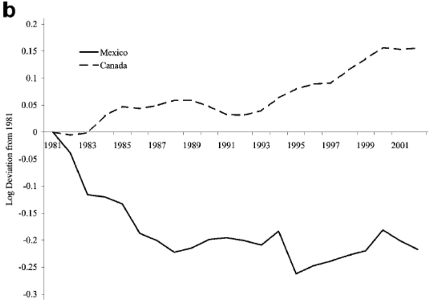


FIG. 1.—GDP per capita and Solow residual. *a*, Annual log GDP per capita for Mexico (solid line) and Canada (dashed line). *b*, Annual log Solow residual for Mexico and Canada. All values are expressed as deviations from 1981. See the Appendix for data sources and construction of the Solow residual.

Model Aguiar-Gopinath 2007

- Period Utility Household (Cobb-Douglas consumption and leisure):

$$U_t = \frac{\left[C_t^{\gamma}\left(1-L_t\right)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$$

- Cobb-Douglas production function with temporary and permanent productivity shocks:

$$Y_t = e^{z_t} K_t^{1-\alpha}\left(\Gamma_t L_t\right)^{\alpha}$$

$$\Gamma_t = \prod_{s=0}^{t} e^{g_t}$$

$$z_t = \rho_z z_{t-1} + \epsilon_t^z \qquad g_t = \left(1-\rho_g\right)\mu_g + \rho_g g_{t-1} + \epsilon_t^g$$

- Budget constraint (with quadratic adjustment cost of investment)

$$C_t + K_{t+1} = Y_t + \left(1-\delta\right)K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - e^{\mu_g}\right)^2 K_t - B_t + q_t B_{t+1}$$

Debt Elastic Interest Rate

- Interest Rate depends on the level of debt (target the level of normalized debt b)

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + \psi \left[exp\left(\frac{B_{t+1}}{\Gamma_t} - b \right) - 1 \right]$$

- Variable need to be de-trended to render them stationary: $\hat{x}_t = x_t / \Gamma_{t-1}$
- The de-trended (and recursive) problem:

$$V = max_{\{\hat{C}, L, \hat{K}', \hat{B}'\}} \frac{\left[\hat{C}_t^{\gamma} (1 - L_t)^{1-\gamma} \right]^{\frac{1-\sigma}{}}}{1 - \sigma} + \beta e^{g\gamma(1-\sigma)} EV'$$

$$\hat{C} + e^g \hat{K}' = \hat{Y} + (1 - \delta) \hat{K} - \frac{\phi}{2} \left(e^g \frac{\hat{K}'}{\hat{K}} - e^{\mu_g} \right)^2 \hat{K} - \hat{B} + e^g q \hat{B}'$$

FOC (without hats) and Steady-State

$$\gamma \frac{U}{c} = \lambda$$

$$(1-\gamma)\frac{U}{1-L} = \alpha \frac{Y}{L}$$

$$\lambda e^{g}\left(1+\phi\left(e^{g}\frac{K'}{K}-e^{\mu}\right)\right) = \beta e^{g\gamma(1-\sigma)}\lambda'[(1-\alpha)\frac{Y'}{K'}+(1-\delta)]$$
$$-\frac{\phi}{2}\left(\frac{e^{g}K''}{K'}-e^{\mu_g}\right)^2+\frac{\phi e^{g}K''}{K'}\left(\frac{e^{g}K''}{K'}-e^{\mu_g}\right)$$

$$\lambda e^{g} q = \beta e^{g\gamma(1-\sigma)}\lambda'\left[1+r+\psi\left(e^{B-b}-1\right)\right]$$

- Steady-state:

$$1 = \beta e^{\mu_g[\gamma(1-\sigma)-1]}\left[(1-\alpha)\frac{Y}{K}+(1-\delta)\right]$$

$$1 = \beta e^{\mu_g[\gamma(1-\sigma)-1]}(1+r)$$

$$C + K\left(e^{\mu_g}+\delta-1\right) = Y + B(e^{g}q-1)$$

Calibration Aguiar and Gopinath

Parameter	Value	Description
β	.98	Time Preferences
γ	.36	Labor equals one-third
b	0.1	Normalized debt
ψ	0.001	Interest Rate Premium
α	0.68	Labor Share
σ	2	Risk Aversion
δ	.5	Depreciation Rate
ϕ	4	Capital Adjustment Cost
$\mu_{\boldsymbol{g}}$	log(1.0066)	Calibrated for Mexico
$\rho_{\boldsymbol{z}}$	0.95	Calibrated for Mexico
$\rho_{\boldsymbol{g}}$	0.01	Calibrated for Mexico
$\sigma_{\epsilon^{\boldsymbol{g}}}$	0.0281	Calibrated for Mexico
$\sigma_{\epsilon^{\boldsymbol{z}}}$	0.048	Calibrated for Mexico

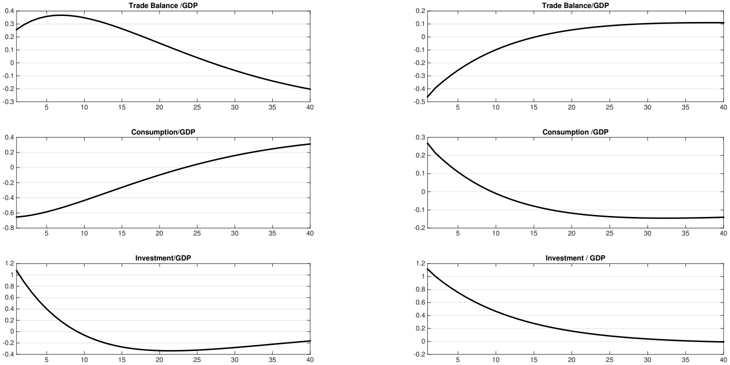
Notes: Source: Aguiar and Gopinath (2007)

Emerging Economy Business Cycles

	Mexico	**Model**
σ_y	2.40	2.39
$\sigma_{\triangle(y)}$	1.52	1.72
σ_c/σ_y	1.26	1.27
σ_i/σ_y	4.15	2.59
$\sigma_{tb/y}/\sigma_y$	0.90	0.71
ρ_y	0.83	0.78
$\rho_{y,tb/y}$	-0.75	-0.66
$\rho_{c,y}$	0.92	0.95
$\rho_{c,y}$	0.91	0.92

Notes: Own calculations and Aguiar and Gopinath (2007)

Impulse-Response Functions

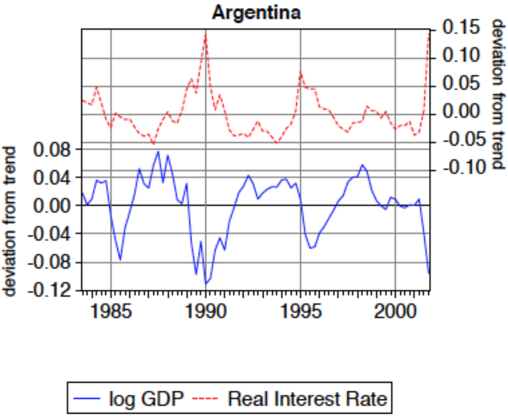


Transitory (z) Permanent (g)

Figure : IRF (permanent and transitorily productivity shock)

The Role of Interest Rate Shocks (Perri-Neumeyer 2005)

- Credit conditions change easily for emerging econonies
- Interest rates are correlated to business cycles



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Model Perri-Neumeyer 2005

- Period Utility Household (Greenwood, Hercowitz, and Huffman (1988) preferences over consumption and leisure):

$$U_t = \frac{\left[c_t - \psi (1+\gamma)^t l_t^{\nu}\right]^{1-\sigma}}{1-\sigma}$$

- Cobb-Douglas production function with with working capital:

$$y_t = A_t k_t^{\alpha} \left((1+\gamma)^t l_t\right)^{1-\alpha}$$

$$\pi_t = y_t - w_t l_t - r_t k_t - [R_t - 1] \theta w_t l_t$$

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_t^A \qquad R_t = R^* D$$

- Budget constraint (with quadratic adjustment cost of investment)

$$c_t + i_t + b_{t+1} + \kappa (b_{t+1}) = w_t l_t + r_t^k k_t + b_t R_t$$

Perri-Neumeyer 2005

- Interest Rate depends on a premium for risk assets R^* (common to all countries) and a country-specific default D

$$R_t = R_t^* D_t \qquad \hat{R}^* = \rho_1 R_{t-1}^* + \epsilon_t^{R^*} \qquad \hat{D}_t = \rho_1 D_{t-1} + \epsilon_t^D$$

- Investment and Portfolio adjustment costs:

$$i_t = k_t - (1-\delta)\, k_{t-1} + \frac{\phi}{2} k_{t-1} \left(\frac{k_t}{k_{t-1}} - (1+\gamma) \right)^2$$

$$\kappa\left(b_{t+1}\right) = \frac{\kappa}{2} y_{t+1} \left(\frac{b_{t+1}}{y_{t+1}} - \bar{b} \right)^2$$

FOC Perri-Neumeyer 2005

$$\left(\hat{c}_t - \psi l_t^{\nu}\right)^{-\sigma} = \lambda$$

$$\psi \nu l_t^{\nu-1} = w_t = (1-\alpha)\,\frac{\hat{y}_t}{l_t}\left(\frac{1}{1+r_t\theta}\right)$$

$$\lambda\left[1+\phi\left(\frac{(1+\gamma)\,k'}{k}-(1+\gamma)\right)\right] = \beta\,(1+\gamma)^{-\sigma}\,\lambda'[(1-\alpha)\,\frac{y'}{k'}+(1-\delta)$$
$$-\frac{\phi}{2}\left((1+\gamma)\,k''/k'-(1+\gamma)\right)^2$$
$$+\frac{\phi k''}{k'}\left(\frac{(1+\gamma)\,k''}{k'}-(1+\gamma)^{\mu_g}\right)]$$

$$\lambda\left[1+\kappa\left(\frac{b'}{y'}-\bar{b}\right)\right] = \beta\,(1+\gamma)^{-\sigma}\,\lambda' R'$$

Interest Rate Shock

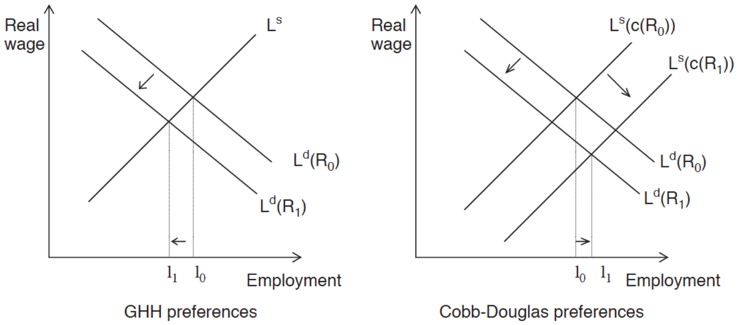

Figure : Labor Markets with GHH and Cobb-Douglas Preferences

Calibration Perri-Neumeyer

Parameter	Value	Description
β	0.93	Time Preferences
σ	5	Risk Aversion
ν	1.6	Labor Disutility
ψ	2.48	Labor Weight
γ	0.0062	Growth Rate
α	0.38	Capital Share
δ	0.044	Depreciation Rate
θ	1	Labor Paid in Advance
κ	10^{-5}	Bond Cost Holding
ϕ	25.1	Adjustment cost Capital
$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_t^A$	$\rho_A = 0.95, \sigma_{\varepsilon^A} = 0.95$	Productivity Shocks
$\hat{R}_t^* = \rho_1 \hat{R}_{t-1}^* + \epsilon_t^R$	$\rho_1 = 0.81, \sigma_{\varepsilon^R} = 0.63\%$	International Rate
$\hat{D}_t = \rho_2 \hat{D}_{t-1} + \epsilon_t^D$	$\rho_2 = 0.78, \sigma_{\varepsilon^D} = 2.59\%$	Country Risk

Source: Neumeyer and Perri (2005)

Impulse-Response Functions

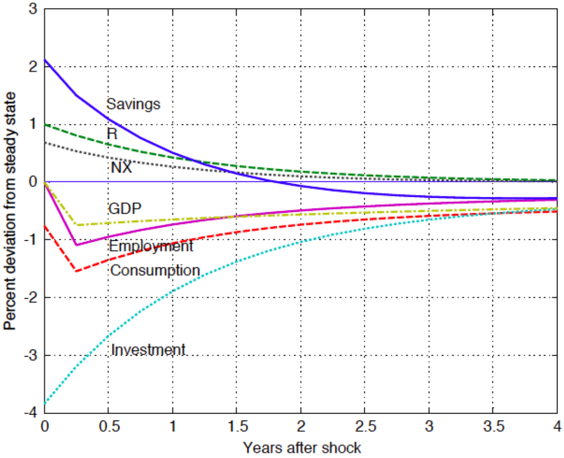

Figure : IRF (Interest Rate Shock)

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