Master in Economics
Lecture 2: Two-Country Models

International Business Cycle

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Two-Country Models

- Two-country models ask whether extensions of closed-economy models that explain domestic business cycles can also help to understand the international dimension of aggregate fluctuations
- The cannonical paper is Backus, Kehoe, and Kydland (1992) that extends Kydland and Prescott (1982) to a two-country setup
- The key question is whether a single aggregate productivity shock can explain at the same time the beviour of domestic and international variables

Two-Country Model (BKK 1992)

- Two countries, each represented by a large number of infitinitely-lived representative household that maximizes consumption (c) and leisure $(1 - l)$ subject to stochastic productivity shocks at home (Z) and abroad (Z^*).
- The production function is Cobb-Douglas

$$max_{\{c_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\mu} (1 - l_t)^{1-\mu}\right]^{1-\sigma}}{1 - \sigma} \quad (1)$$

$$y_t = Z_t k_t^{\alpha} l_t^{1-\alpha} \quad (2)$$

$$nx_t = y_t - c_t - k_{t+1} + (1 - \delta) k_t \quad (3)$$

The Planner's Problem

- The planner's problem is the maximization of the weigthed sum (Pareto-Negishi Ψ) of each country's utility subject the production functions and the aggregate resource constraint:

$$Max_{\left\{c_t, c_t^*, l_t, l_t^*, k_{t+1}, k_{t+1}^*\right\}} \left[\Psi U\left(c_t, l_t\right) + \left(1-\Psi\right) U\left(c_t^*, l_t^*\right)\right] \quad (4)$$

$$y_t + y_t^* = c_t + c_t^* + i_t + i_t^* \quad (5)$$

- The planner equalizes marginal utility of consumption:

$$\Psi U_c\left(c, l\right) = \left(1-\Psi\right) U_c\left(c^*, l^*\right)$$

- The other FOC are standard:

$$\frac{\left(1-\mu\right)}{\mu} \frac{c}{1-l} = \left(1-\alpha\right) \frac{y}{l}$$

$$\lambda = \beta \lambda' \left(\alpha \frac{y'}{k'} + 1 - \delta\right)$$

Decentralized Problem (Home)

- This problem can be decentralized by assuming there is a full set of state-contigent claims (Arrow-Debrew securities in sequential markets)
- An event $s_t \in S$ is a realization of productivities (potentially infinite set). A history is a collection of events: $s^t = (s_0, .., s_t)$.
- The optimization problem of the domestic household is

$$Max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \frac{\left[c\left(s^t\right)^{\mu}\left(1-l\left(s^t\right)\right)^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \qquad (6)$$

$$\begin{aligned} w\left(s^t\right) l\left(s^t\right) + r\left(s^t\right) k\left(s^t\right) + b\left(s^t\right) = {} & c\left(s^t\right) + k\left(s^{t+1}\right) \\ & + (1-\delta) k\left(s^t\right) \\ & + \sum_{s_{t+1}} q\left(s^t, s_{t+1}\right) b\left(s^t, s_{t+1}\right) \end{aligned}$$

Descentralized Problem (Foreign)

- The foreign household maximizes

$$Max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \frac{\left[c\left(s^t\right)^{*\mu}\left(1-l^*\left(s^t\right)\right)^{1-\mu}\right]^{1-\sigma}}{1-\sigma} \tag{7}$$

- Subject to the following budget constraint:

$$\begin{aligned} & w^*\left(s^t\right) l^*\left(s^t\right) \\ +r^*\left(s^t\right) k^*\left(s^t\right) + b^*\left(s^t\right) &= c^*\left(s^t\right) + k^*\left(s^{t+1}\right) \\ & + (1-\delta) k^*\left(s^t\right) \\ & + \sum_{s_{t+1}} q\left(s^t, s_{t+1}\right) b^*\left(s^t, s_{t+1}\right) \end{aligned}$$

Optimal Conditions

$$\lambda\left(s^t\right) = U_c\left(s^t\right) \tag{8}$$

$$q\left(s^t, s_{t+1}\right) = \beta \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \tag{9}$$

$$1 = \sum_{s_{t+1}} \beta \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{\lambda\left(s^t, s_{t+1}\right)}{\lambda\left(s^t\right)} \left[r\left(s^t\right) + 1 - \delta\right] \tag{10}$$

$$\frac{1-\mu}{\mu} \frac{c\left(s^t\right)}{1 - l\left(s^t\right)} = w\left(s^t\right) \tag{11}$$

$$\frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} = \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c^*\left(s^t\right)} \tag{12}$$

Calibration BKK Model

Parameter	Value	Description
β	0.99	Time Preferences
μ	0.34	Consumption Share
σ	2	Intertemporal Elasticity
α	0.36	Capital Share
δ	0.5	Depreciation Rate

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = A \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}$$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_2 \end{bmatrix}$	$\begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$	Productivity Shocks
$\rho_{\epsilon,\epsilon^*}$	0.258	Correlation Shocks
$\sigma_\epsilon^2 = \sigma_{\epsilon^*}^2$	0.00852	Variance Shocks

Notes: Backus, Kehoe, and Kydland (1992)

Impulse Response Functions

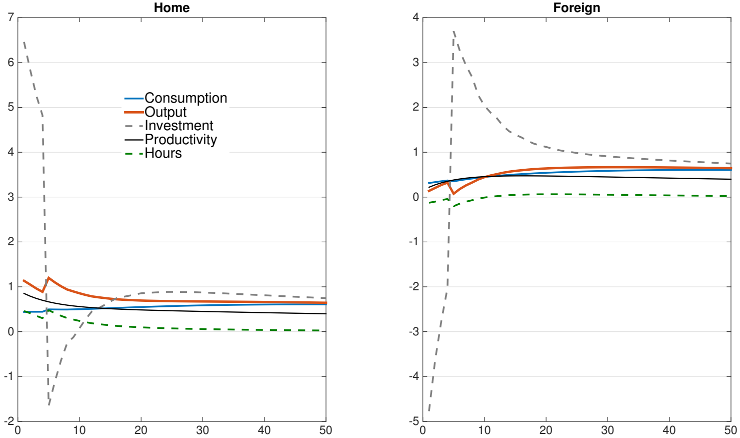


Figure : IRS (percentage deviations of steady-state)

Business Cycles (Domestic Variables)

	σ_x/σ_{GDP}	$\rho_{x,GDP}$	σ_x/σ_{GDP}	$\rho_{x,GDP}$	σ_x/σ_{GDP}	$\rho_{x,GDP}$
GDP	1	1	1	1	1	1
C	0.75	0.82	0.83	0.81	0.42	0.77
I	3.27	0.94	2.09	0.89	10.99	0.27
L	0.61	0.88	0.85	0.32	0.50	0.93
Z	0.68	0.96	0.98	0.85	0.67	0.89
nx	0.27	-0.37	0.49	-0.25	2.51	0.01

Notes: Backus, Kehoe, and Kydland (1995)

Business Cycles (International Co-movement)

	US and Europe	Model
	International Correlations	
GDP	0.66	-0.21
C	0.51	0.88
I	0.53	-0.94
L	0.33	-0.78
Z	0.56	0.25

Notes: Backus, Kehoe, and Kydland (1995)

BKK (1995)

- Each country specializes in the production of intermediate goods (a for home, b for foreign):

$$a_{1t} + a_{2t} = y_t = Z_t k_t^{\alpha} l_t^{1-\alpha}$$

$$b_{1t} + b_{2t} = y_t^* = Z_t^* k_t^{*\alpha} l_t^{*1-\alpha}$$

- The two goods are aggregated using an Armington Aggregator with elasticity of substitution $1/\theta$

$$c_t + i_t = \left[\omega a_{1,t}^{1-\theta} + (1-\omega)\, b_{1,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

- If p_{1t}^a and p_{1t}^b are the prices of domestic and foreign goods in units of domestic final good, the terms of trade are: $tot_t = \frac{p_{1t}^a}{p_{1t}^b} = \left(\frac{a_{1t}}{b_{1t}}\right)^{\theta} \frac{1}{\omega}$
- The trade balance to GDP ratio and the RER are:

$$nx_t = \frac{p_{1t}^a a_{2t} - p_{1t}^b b_t}{y_t} \qquad rer_t = \frac{p_{1t}^a}{p_{2t}^a} = \frac{p_{1t}^b}{p_{2t}^b}$$

Calibration Heathcote and Perri (2002)

Parameter	Value	Description
β	0.99	Time Preferences
μ	0.34	Consumption Share
σ	2	Intertemporal Elasticity
α	0.36	Capital Share
δ	0.5	Depreciation Rate
is	0.15	Import Share
$1/\theta$	0.9	Elasticity of Substitution
$\rho_{\epsilon,\epsilon*}$	0.290	Correlation Shocks
$\sigma_\epsilon = \sigma_{\epsilon*}$	0.0073	Variance Shocks
$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_2 \end{bmatrix}$	$\begin{bmatrix} 0.970 & 0.025 \\ 0.025 & 0.970 \end{bmatrix}$	Productivity Shocks

Notes: Heathcote and Perri (2002)

Business Cycles (two-goods BKK model)

	Data			Model		
	σ_x/σ_{GDP}	$\rho_{x,GDP}$	ρ_{x,x^*}	σ_x/σ_{GDP}	$\rho_{x,GDP}$	ρ_{x,x^*}
GDP	1	1	0.58	1	1	0.18
C	0.81	0.86	0.36	0.53	0.96	0.65
I	2.84	0.95	0.30	2.74	0.96	0.29
L	0.66	0.87	0.42	0.31	0.97	0.14
nx	0.27	-0.49		0.43	-0.64	
tot	1.79	-0.24		0.61	0.65	
rer	4.02	0.13		0.45	0.65	

Notes: Heathcote and Perri (2002)

The Role of the Elasticity of Substitution

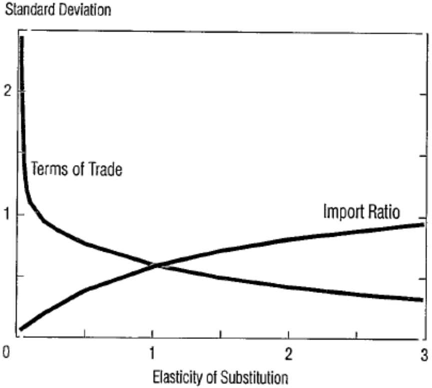


Figure : Effects of Varying the Elasticity of Substitution on the Volatility of TOT and Import Ratio

Notes: Backus et al. (1995)

The Role of the Financial Structure

- Heathcote and Perri (2002) study the effects of departing from the complete market assumptions: bond market economy and financial autarky
- The only element that changes in the set up of the problem is the budget constraint of households
- Bond Economy:

$$p_1^a\left(s^t\right)\left[w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^t\right)\right]=c\left(s^t\right)+i\left(s^t\right)$$
$$+p_1^a\left(s^t\right)b\left(s^t\right)$$
$$+\,p_1^a\left(s^t\right)q\left(s^t\right)b\left(s^{t+1}\right)$$

- Financial Autarky:

$$p_1^a\left(s^t\right)\left[w\left(s^t\right)l\left(s^t\right)+r\left(s^t\right)k\left(s^t\right)\right]=c\left(s^t\right)+i\left(s^t\right)$$

Equilibrium Conditions under different financial structures

- Complete Markets

$$q\left(s^t\right) = \beta \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{p_1^a\left(s^t, s_{t+1}\right)}{p_1^a\left(s^t\right)}$$

$$rer\left(s^t, s_{t+1}\right) = k \frac{U_c^*\left(s^t, s_{t+1}\right)}{U_c\left(s^t, s_{t+1}\right)}$$

where $\kappa = rer\left(s_0\right) U_c\left(s_0\right) / U_c^*\left(s_0\right)$

- Bond Economy

$$q\left(s^t\right) = \frac{1}{1 + r\left(s^t\right)} = \beta \sum_{s_{t+1}} \frac{\pi\left(s^t, s_{t+1}\right)}{\pi\left(s^t\right)} \frac{U_c\left(s^t, s_{t+1}\right)}{U_c\left(s^t\right)} \frac{p_1^a\left(s^t, s_{t+1}\right)}{p_1^a\left(s^t\right)}$$

$$b\left(s^t\right) = b^*\left(s^t\right)$$

- Financial Autarky: $nx_t = 0$

Business Cycle Statistics (Domestic)

(A) Volatilities[a]

Economy	% std. dev.	% std. dev. / % std. dev. of y			% std. dev.			
	y	c	x	n	ex	im	nx	ir
US data	1.67	0.81	2.84	0.66	3.94	5.42	0.45	4.07
Complete markets	1.21	0.53	2.74	0.31	0.99	0.99	0.20	0.70
Bond economy	1.21	0.52	2.73	0.32	0.96	0.96	0.19	0.76
Financial autarky	1.18	0.51	2.04	0.28	1.29	1.18	0.00	1.51

(B) Correlations with output[b]

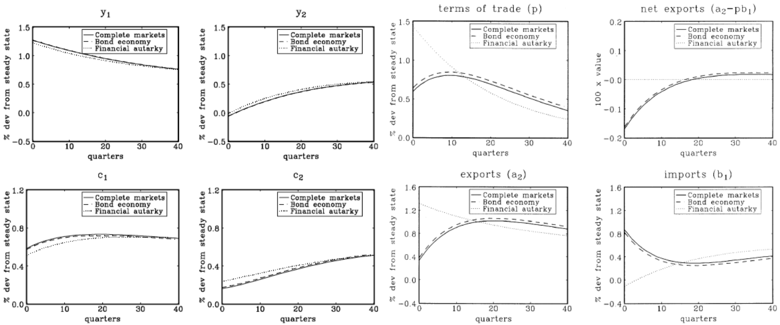
Economy	correlation between c, y	x, y	n, y	ex, y	im, y	nx, y	p, y	rx, y
US data	0.86	0.95	0.87	0.32	0.81	− 0.49	− 0.24	0.13
Complete markets	0.96	0.96	0.97	0.55	0.89	− 0.64	0.65	0.65
Bond economy	0.95	0.96	0.97	0.59	0.86	− 0.65	0.65	0.65
Financial autarky	0.92	0.99	0.99	1.00	0.15	0.00	0.65	0.65

Business Cycle Statistics (Cross-Country)

(C) Cross country correlations and international relative price volatility

Economy	correlation between y_1, y_2	c_1, c_2	x_1, x_2	n_1, n_2	% std. dev. p	rx
Data	0.58	0.36	0.30	0.42	2.99	3.73
Complete markets	0.18	0.65	0.29	0.14	0.78	0.55
Bond economy	0.17	0.68	0.29	0.17	0.84	0.59
Financial autarky	0.24	0.85	0.35	0.14	1.68	1.18

Impulse Response Functions



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