Asset Prices in Production Economies with an Endogenous Extensive Margin

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Abstract

In economies that exhibit love-for-variety either in preferences or technology, the official consumer price index differs from the welfare-based price aggregator. This wedge implies that the stochastic discount factor should be adjusted to account for changes in the range of goods. This paper shows that a real business cycle model with endogenous entry fits aggregate quantities while featuring a higher risk premium when changes in growth variety are properly accounted. Moreover, adding the growth rate of establishments, as a proxy for growth variety, helps the standard consumption-based model to deliver lower pricing errors for the cross-section of stock returns.

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1. Introduction

In economies in which households exhibit love-for-variety preferences or final goods are CES composites of intermediate varieties, aggregate welfare is a function of both the quantity and the range of consumer goods.[2] However, the welfare gains or losses associated to changes in the number of varieties are imperfectly gauged by real consumption measures because the official Consumer Price Index (CPI) doesn't capture the marginal utility of new varieties as the theoretical price aggregator does.[3] This wedge between theoretical and measured variables implies

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[2]Since the work of Dixit and Stiglitz (1977) and Spence (1976) many economic models incorporate preferences or production technologies that aggregate final or intermediate goods using a Constant Elasticity of Substitution (CES) function.

[3]This mismeasurement problem arise because statistical agencies aggregate individual prices at the product level without guidance from a theoretical aggregator. The CPI is just an approximation to the cost-of-living-index (COLI), which in the case of love-for-variety determines how changes in the set of consumed good impact consumer welfare. Additionally, there is imperfect sampling in changes in the quality and quantity of varieties when measuring the CPI. Confronted with new products at the sampling stage, statistical offices only incorporate new varieties once the product reappears in the sample. Furthermore, new products may be introduced in the sample with a long delay. See Chapter 17 of the BLS Handbook of Methods for a discussion of the differences between the CPI and the COLI

that the stochastic discount factor derived from the standard model under the homogenous good assumption needs to be adjusted when the set of goods changes over the business cycle in order to fully account for the true aggregate risk faced by households.[4]

This paper discusses the role of changes in the set of goods for asset pricing in the context of a business cycle model with love-for-variety preferences and production at both the intensive and extensive margin. I find that the model, while fitting the second moments of real quantities, matches the sharpe ratio of the stock market excess returns in the US and yields an annual equity premium close to 1%. In this economy, the stochastic discount factor exhibits greater volatility as the number of varieties fluctuates over the cycle and commoves with the growth rate of real consumption, which alone doesn't capture the exposure of households to macroeconomic risk.

The model is the closed-economy version of Fattal Jaef and Lopez (2014) and similar to the environment studied by Ghironi and Melitz (2005), Alessandria and Choi (2007) and Bilbiie et al. (2012). Intermediate firms, which operate a Cobb-Douglas production function combining labor and physical capital, produce different varieties aggregated by a CES function. Firm-level productivity is composed of an aggregate component, which evolves stochastically over time, and an idiosyncratic component, which is fixed during the lifetime of the firm. An infinite mass of potential entrants can become producers after paying a sunk cost of entry denominated in effective units of labor. There are fixed costs of production, so not all intermediate producers operate in each period.

The baseline model delivers a much higher risk premium than the standard RBC model while featuring realistic dynamics for aggregate quantities and pro-cyclical establishment entry, which is consistent with the findings by Chatterjee and Cooper (1993), Alessandria and Choi (2008) and Lee and Mukoyama (2012). The volatility of real consumption in the model, properly deflated and hence comparable to real consumption in the data, is higher than the volatility of nominal consumption in final good units. A positive aggregate productivity shock induces higher output and consumption at the intensive margin but also encourages labor reallocation towards the creation of new varieties. As new entrants become operative the number of total varieties increases. Altogether, consumption in units of the final good remains fairly stable, as dictated by the Euler equation of the model, while real consumption, which excludes the extensive margin, exhibits richer dynamics governed by the tension between the intensive and the extensive margin of production.[5]Different from the existing literature, the model here discussed delivers a higher risk premium without departing from the standard time-separable power utility and without capital or labor frictions.[6]

The model is calibrated to match key features of the US economy at a quarterly frequency and for the baseline specification I choose a value of 3.5 for the elasticity of substitution of

[4]The wedge between the theoretical and the measured consumer price index can be large as documented by a growing literature devoted to study the extensive margin of production and trade. Broda and Weinstein (2006) find an annual upward bias for the price index of import varieties in the US of 1.2 percentage points since 1972 to 2001, which amounts to unmeasured welfare gains equal to 2.6% of GDP. Using a large dataset for Japan, Weinstein et al. (2013) report a measurement bias of the CPI that varies depending on the level of measured inflation and ranges from -0.8 to -1.8 percentage points per year. Mohler and Seitz (2012) document large unmeasured welfare variety gains within the European union. Ghironi and Melitz (2005), Alessandria and Choi (2007) and Alessandria and Choi (2008) discuss the discrepancy between theoretical and measured price indices when discussing models with an extensive margin of production.

[5]The solution technique is similar to Jermann (1998). Standard RBC statistics are computed using a first-order approximation method while asset pricing statistics are computed using lognormal formulae based on an second-order approximation.

[6]Examples of general equilibrium models with frictions to capital or labor mobility that explain asset pricing facts are: Jermann (1998), Boldrin et al. (2001) and Massimiliano Croce (2014).

intermediate goods. This parameter is key for the asset prices predictions of the model as it governs the degree of curvature of the CES aggregator. I set this parameter so as to match steady-state mark ups and thus discipline the asset pricing predictions of the model by real quantities. Under a parameterization in which the elasticity of substitution is equal to 2.5, the model delivers a higher sharpe ratio than the one observed in the data and overshoots the equity premium, but at the cost of counterfactually exhibiting high steady-state mark-ups and excessive volatility of aggregate quantities. On the contrary, a high value for the elasticity of substitution, consistent with small mark-ups, makes the effects on changes in the number of firms disappear, and with them most of the novel asset pricing predictions of the model.

The calibration also considers a specification of the entry cost function that allows the model to match the volatility of new establishments in the US at annual frequency. Theoretically, the extensive margin of the model should be calibrated to match cyclical changes in the number of varieties in the economy. Lacking such a measure, I use as a proxy the number of establishments as reported by the Business Dynamic Statistics (BDS) for the period 1977-2011. As many establishments are multi-product, using this variable cannot fully solve the mismeasurement problem of consumer welfare. However this measure is arguably the best available proxy and it is close to the ideal metric when discussing some sectors of the economy, like services.

Finally, I test whether the asset pricing predictions of the model have empirical support beyond the fit of the model. With this goal I compute the pricing errors for the cross-section of stock returns of the standard consumption-based model (CCAPM) augmented with different proxies for growth variety. I find that using the establishment entry rate as a pricing factor delivers lower cross-sectional pricing errors than the CCAPM and than the benchmark three Factor Model for the Fama and French's portfolios sorted by industries. Similar results hold when using other proxies from the BDS such as the establishment growth rate or the share of jobs created by new establishments. As a robustness I also use the growth rate of the number of firms or patent applications, which are available for a longer sample period.

The rest of the paper is organized as follows. Section 2 discusses related papers in the literature and details how this paper differs from previous work. Section 3 describes the benchmark model. Section 4 explains the measurement issues of the model and the required adjustment needed to compare real quantities with their data counterparts. Section 5 presents the quantitative results of the general equilibrium model and the sensitivity analysis. Section 6 discusses the empirical asset pricing implications of the model when the number of varieties are properly accounted. The last section concludes.

2. Related Literature

Secular trends in the extensive margin of production are at the base of endogenous growth models with product innovations such as as Grossman and Helpman (1991). Melitz (2003) studies the effect of intra-industry reallocation after a reduction in trade barriers in a model with extensive margin of production. A more recent line of research studies the role of the extensive margin of production as a propagation mechanism for real business cycles. Ghironi and Melitz (2005), Alessandria and Choi (2007), Alessandria and Choi (2008) and Fattal Jaef and Lopez (2014) discuss the role of cyclical changes in the extensive margin of production for international business cycles, while Bilbiie et al. (2012) study their impact on aggregate quantities in a closed-economy set up.

This paper shares with this literature the basic modeling framework and the discussion of the measurement problems present in models with an extensive margin of production, in particular the wedge between the theoretical price index and the consumer price indices computed by statistical offices. This paper expands this line of research by focusing on the joint dynamics of real quantities and asset prices in economies with production both at the intensive and extensive margins.

Recently Scanlon (2008), Bidian et al. (2013) and Loualiche (2014) have also argued that adding an extensive margin of production can improve the asset pricing implications of the standard model. This paper differs from their work as it studies the role of the extensive margin of consumption growth in a general equilibrium setup calibrated to account for key features of the US economy. In addition, this paper evaluates the empirical pricing predictions of a model that exhibits love-for-variety by using a direct measure of the number of establishments for the US economy, as a proxy for growth variety. Scanlon (2008) evaluates the effects of changes in the number and quality of goods for asset prices but doesn't study the general equilibrium effect on aggregate quantities. Bidian et al. (2013) study a similar question but focus on the estimation of the risk aversion of the Euler equation in a multi-sector model, while Loualiche (2014) highlights the role of the aggregate cost of entry for asset prices.

Earlier papers by Yogo (2006) and Aït-Sahalia et al. (2004) study the asset pricing effects of specific consumption bundles, either durables and non-durables or luxury goods, and discuss the covariance between these bundles and market excess returns. This paper is silent about the composition of these consumption bundles, but highlights the welfare effects of changes in the total number of goods and their correlation with consumption and stock market returns.

3. Base Model

The model economy is composed of a representative household, competitive producers of a final good and a continuum of heterogeneous producers of intermediate goods that operate under monopolistic competition.

3.1. Households

Households maximize their expected discounted sum of flow utility given by consumption (C) and the disutility associated to supplying labor (L). Preferences on consumption are of the CRRA family with a discount factor β and risk-aversion parameter γ, which also governs the inter-temporal elasticity of substitution.

$$\max_{C_t, L_t, K_{t+1}, B_{t+1}, \varphi_{t+1}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} - A \frac{L_t^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}}$$

Households exhibit a constant Frisch Elasticity of Labor Supply (ϵ) and use their labor income to either consumer or invest. Households have access to three type of assets: physical capital, shares of a mutual fund that owns all firms in the economy and one-period non-contingent bond. The budget constraint in units of consumption can be written as:

$$C_t + K_{t+1} + \varphi_{t+1}\tilde{v}_t \left(N_{D,t} + N_{e,t}\right) + B_{t+1} = R_{k,t}K_t + (\tilde{d}_t + \tilde{v}_t)\varphi_t N_{D,t} + w_t L_t + R_{f,t}B_t$$

Letting φ_t be the shares on the domestic mutual fund held by domestic households, the total amount of resources saved in shares is $\varphi_{t+1}\left(N_{D,t} + N_{e,t}\right)\tilde{v}_t$, where $\left(N_{D,t} + N_{e,t}\right)$ is the number of firms owned by the mutual fund and $\tilde{v}_t$ is the average value in units of the home final good.

Households receive next period dividends $\tilde{d}_{t+1}$ from a fraction $(1-\delta)$ of the $(N_{D,t}+N_{e,t})$ firms in the mutual fund, and have the option to sell shares at a value of $\tilde{v}_{t+1}N_{D,t+1}$.

The optimal conditions that characterize the solution to the household's problem include a labor-leisure equation:

$$AL_t^{\frac{1}{\epsilon}} = C_t^{-\gamma} w_t$$

and three Euler equations for the returns on physical capital, shares of the mutual fund and the non-contingent bond.

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^i_{t+1} \right] \tag{1}$$

where R^i_{t+1} denotes the gross return of any of the asset, namely: capital, $R_{k,t+1} = 1 + r_{k.t+1} - \delta$, equity markets, $R_{E,t+1} = (1-\delta_N)\frac{\tilde{v}_{t+1}+\tilde{d}_{t+1}}{\tilde{v}_t}$, or the non-contingent bond, $R_{f,t}$

3.2. *Firms*

3.2.1. *Producer of Final Goods*

In every period there is a set of intermediate goods Ω_t that are available for the production of the final good. Let $z \in \Omega$ be a particular variety. The producer of the final good combines intermediate inputs according to:

$$Y_t = \left[\int_{z\in\Omega_t} y_t(z)^{\frac{(\theta-1)}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

Here θ denotes the elasticity of substitution among varieties and $y_t(z)$ the demand for the domestic variety z.

Let P_t be the price of the final good, taken as given by the competitive producer, and let $p(z)$ denote the price of a domestic variety z. The problem of the producer of final goods is given by:

$$\max_{y_t(z)} P_t Y_t - \int_{z\in\Omega_t} p_t(z) y_t(z) dz$$

subject to the technology in (2). The solution to this problem gives the following demand function:

$$y_t(z) = \left(\frac{p_t(z)}{P_t} \right)^{-\theta} Y_t \tag{3}$$

and the aggregate price level:

$$P_t = \left[\int_{z\in\Omega_t} p(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{4}$$

Finally, since the final good is either consumed or invested, the aggregate resource constraint is defined by $Y_t = C_t + I_t$, where $I_t = K_{t+1} - (1-\delta)K_t$.[7]

3.2.2. Producers of Intermediate Goods

Firms are differentiated by their productivity, so there is no loss of generality in indexing by z.[8]At any point in time there exists a mass $N_{D,t}$ of firms that produce and sell in domestic markets. These firms act under monopolistic competition, so they take the demand functions for their variety as given. They operate a Cobb Douglas production function technology with identical factor shares across varieties:

$$Y_t(z) = Z_t z\,[k_t(z)]^{\alpha}\,[l_t(z)]^{1-\alpha} \tag{5}$$

where $k_t(z)$ and $l_t(z)$ represent the demand for physical capital and labor. Heterogeneity is reflected in differences in the TFP component of each firm z, which I assume is fixed along the firm's lifetime. All firms are subject to a common aggregate productivity factor Z_t, which evolves stochastically in the model .

Conditional on entry, firms decide to engage actively in production after paying a per-period fixed cost f, measured in effective units of labor. Because of this fixed costs, only a subset of firms operate during each period. Given these definitions, the optimization problem of an intermediate producer is described by equation (6) subject to (7) and (8).

$$\max \rho_t(z)y_t(z) - w_t l_t(z) - r_{k,t}k_t(z) - x(t)\frac{w_t}{Z_t}f \tag{6}$$

$$y_t(z) = Z_t z\,[k_t(z)]^{\alpha}\,[l_t(z)]^{1-\alpha} \tag{7}$$

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} Y_t \tag{8}$$

The term $x(t)$ is an indicator function that captures whether the firm is active or not. Letting $MC_{p,t}(z)$[9] denote the marginal cost of production, the optimal pricing rule is defined by:$\rho_t(z) = \frac{\theta}{\theta-1}MC_{p,t}(z)$, which is the familiar result for problems with monopolistic competition and aggregate CES technology. The demand for factors of production is determined by:

$$k_t(z)r_t^k = MC_{p,t}(z)\alpha\frac{Y_t(z)}{r_{k,t}} \tag{9}$$

[7]In a recent review of the National and Income Product Accounts the BEA has redefined its classification of expenditures on Research and Development (R&D), which according to the new framework will be counted as part of investment. The revised time series for investment is currently available only since 2000, so in order to be consistent with the current historical data, I exclude entry costs as part of investment. In the online appendix I present the main statistics of the model when entry costs are capitalized, which implies redefining investment as: $I_t = K_{t+1} - (1-\delta)K_t + \tilde{v}_t N_{e,t}$

[8]Although I refer to the production units in the model as firms, these do not correspond exactly to what firms are in reality, where a firm may have different products. Implicitly, thus, I am imposing in the model that a firm is equivalent to a variety. For the calibration of the model and the empirical test of the theory, I will use establishments as the relevant metric.

[9]With Cobb Douglas production function and labor share $1-\alpha$, the marginal cost is of the form $\frac{r_K^{\alpha}w^{1-\alpha}}{Zz}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}$, where r is the rental rate of capital stock and w is the wage.

$$l_t(z)w_t = MC_{p,t}(z)(1-\alpha)\frac{Y_t(z)}{w_t} \tag{10}$$

Profits from domestic sales, $d_{D,t}(z)$, are:

$$d_{D,t} = \frac{1}{\theta}\left[\rho_t(z)\right]^{1-\theta} Y_t - x(t)\frac{w_t}{Z_t}f \tag{11}$$

It follows that more productive firms set lower prices and earn higher profits. This is a feature of CES demand functions, which exhibit elasticities with respect to price that are greater than those in absolute value ($\theta > 1$). Fixed costs determine the cut-off level of productivity at which firms operate. Because the relationship between profits and productivity is monotonic, the production cut-off rule takes the following form:

$$\bar{z}_{D,t} = \inf\{z : d_{D,t}(z) \geqslant 0\} \tag{12}$$

where all firms whose productivity is greater than or equal to the cutoff productivity $z_{D,t}$ decide to produce. Provided the lowest productivity firm is sufficiently low to ensure that $z_{D,t}$ is in the interior of the support of the distribution, there always exists a subset of firms operating in the market.

3.2.3. Entry, Exit and Ownership of Firms

Entry is modeled following Melitz (2003). There is an infinite pool of forward looking potential entrants that consider paying a sunk entry cost $\hat{f}_e$ of effective units of labor to get a productivity draw z from a common known distribution $G(z)$. The idiosyncratic productivity is constant over the lifetime of the firm, which can be interrupted at any time with exogenous probability δ. Because productivities are drawn after payment of the entry cost, prospective entrants consider the average value of a firm in making their entry decisions. This is determined by the expected present value of average total profits and must be equal to the entry cost in equilibrium:

$$\hat{f}_e\frac{w_t}{Z_t} = \tilde{v}_t \tag{13}$$

Here $\tilde{v}_t = E_t \sum_{\tau=t+1}^{\infty}\left[\beta(1-\delta)\right]^{\tau-t}\frac{U_c(c_\tau,l_\tau)}{U_c(c_t,l_t)}\tilde{d}_{D,\tau}$ denotes the present value of the expected stream of average profits $\tilde{d}_t$, discounted using the the stochastic discount factor of households augmented to account for the exogenous probability of exit δ. This representation of entry costs implies that the entry technology is subject to productivity shocks, which is a key assumption to generate pro-cyclical entry. The entry cost function $\hat{f}_e$ is defined as follows:

$$\hat{f}_e = f_e + \zeta_e\left[\exp\left(N_{e,t} - \bar{N}_e\right) - 1\right] \tag{14}$$

The parameter ζ_e governs directly the volatility of entry and penalizes deviation of the mass of entrants relative to its steady-state. A very large value of ζ_e reduces entry up to the point that effectively there is no net entry along the business cycle. Indeed, when assuming a large ζ_e and fixing the production threshold $\bar{z}_{D,t}$ the model exhibits a constant number of firms. In the calibration of the model, the parameter ζ_e is set to match the volatility of entry using the BDS data.[10]

Entrants become operative with a one-period lag, so the mass of producers is a state variable in the model and evolves according to:

$$N_t = (1 - \delta_N)(N_{t-1} + N_{e,t-1}) \tag{15}$$

3.3. Aggregation

It is convenient to assume that the underlying distribution of productivities follows a Pareto distribution in order to simply the aggregation of the model. This assumption induces a size distribution of firms that is Pareto, which is a good approximation for the firm size distribution in the US. I assume that the support of the distribution $G(z)$ is $[z_{min}, \infty)$ with $z_{min} = 1$. As in Melitz (2003), there is a summary statistic of the heterogeneity of the model which can be written as:

$$\tilde{z}_{D,t} = \left[\frac{1}{1 - G(\bar{z}_{D,t})} \int_{\bar{z}_{D,t}}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}} \tag{16}$$

This summary statistic can be further simplify thanks to the assumptions regarding the support and the shape of the distribution of idiosyncratic productivities and can be re-written as: $\tilde{z}_{D,t} = \nu$, where $\nu = \left[\frac{\kappa}{\kappa-\theta+1}\right]^{\frac{1}{\theta-1}}$ and κ governs the dispersion of productivities of the Pareto distribution. The mass of active producers is determined by: $N_{D,t} = 1 - G(\bar{z}_{D,t}) N_t = \bar{z}_{D,t}^{-\kappa} N_t$

Using the summary statistic, all the variables of model can be rewritten in terms of average variables. The aggregate price index, for instance, can be written as:

$$P_t^{1-\theta} = N_t \int_{\bar{z}_{D,t}}^{\infty} p_t(z)^{1-\theta} dG(z) = N_{D_t} \tilde{p}_t^{1-\theta} \tag{17}$$

4. Measurement Issues

A well-established property of CES production functions, as the one used here, is that they exhibit love for variety. As the economy experiences entry and exit of intermediate varieties, the aggregate price index fluctuates even if the prices of existing varieties remain constant. Broda and Weinstein (2006) and Weinstein et al. (2013) present evidence suggesting that this love for variety effect goes mostly unmeasured in consumer price indices, which are calculated using a fixed set of goods. In order to compare the model to the data, it is necessary to construct a price index that resembles the one calculated by statistical offices, which only account for price changes of a fix set of existing varieties.

[10]This representation of entry costs is not completely arbitrary. Other papers that deal with endogenous entry also allow for some curvature of the entry cost function, which can be understood as representation of diminishing quality in managerial ability or congestion effects at firm or variety creation. See Fattal Jaef and Lopez (2014) for a discussion on this modeling choice.

Ghironi and Melitz (2005), Alessandria and Choi (2007) and Alessandria and Choi (2008), have suggested an alternative definition to the theoretical price index that is not directly affected by changes in the number of available varieties but it is only a function of wages and the return on physical capital:

$$\bar{P}_t = \left[\int_{\tilde{z}_D}^{\infty} p_t(z)^{1-\theta} \, dG(z) \right]^{\frac{1}{1-\theta}} = \frac{\theta}{\theta - 1} \left(r_t^k / \alpha \right)^{\alpha} (w_t / 1 - \alpha)^{1-\alpha} \, v \frac{\tilde{z}_{D,t}}{Z_t} \tag{18}$$

This data-consistent price index isolates the effect of changes in the number of varieties relative to the model's price index:

$$N_{D,t}^{\frac{1}{1-\theta}} \bar{P}_t = P_t \tag{19}$$

In order to compare real variables in the model to their data counterparts, nominal variables should be deflated by this data-consistent price index as follows:

$$Y_t^R = N_{D,t}^{\frac{1}{1-\theta}} Y_t \quad C_t^R = N_{D,t}^{\frac{1}{1-\theta}} C_t \quad \tilde{v}_t^R = N_{D,t}^{\frac{1}{1-\theta}} \tilde{v}_t \quad \tilde{d}_t^R = N_{D,t}^{\frac{1}{1-\theta}} \tilde{d}_t \tag{20}$$

4.1. The Consumption-Variety Stochastic Discount Factor

Because the theoretical welfare-based variables have to be corrected to make them comparable with measured variables in the data that ignore the changes in the set of varieties, the Euler Equation for equity returns, or for any other asset in the model, should be rewritten as:

$$1 = \beta E_t \left[\left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\gamma} \left(\frac{N_{D,t+1}}{N_{D,t}} \right)^{\frac{1-\gamma}{\theta-1}} R_{E,t+1}^R \right] \tag{21}$$

where $R_{E,t+1}^R = (1 - \delta_N) \frac{\tilde{v}_{t+1}^R + \tilde{d}_{t+1}^R}{\tilde{v}_t^R}$ as indicated by equation (20).[11]If the number of goods is constant, this pricing equation converges to the standard consumption-based asset pricing model (CCAPM). Otherwise, the number of consumed varieties affects the marginal utility and the stochastic discount factor of households.

One can re-write the previous equation using the following notation:

$$1 = E_t \left[m_{t+1} R_{E,t+1}^R \right] \tag{22}$$

[11]In the case of Epstein and Zin (1989) preferences, the growth rate of the number of varieties also appears in the stochastic discount factor. In that case the stochastic discount factor is: $m_{t+1} = \beta^{\frac{\rho-\gamma}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{\gamma-1}{1-\rho}} (R_{W,t+1})^{\frac{\rho-\gamma}{1-\rho}}$ where γ governs the risk aversion, ρ is the inverse of the inter-temporal elasticity of substitution and $R_{W,t+1}$ is the return on wealth. After the required adjustment for the unmeasured welfare changes associated to changes in the number of varieties the discount factor becomes: $m_{t+1} = \beta^{\frac{\rho-\gamma}{1-\rho}} \left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\rho \frac{\gamma-1}{1-\rho}} \left(\frac{N_{D,t+1}}{N_{D,t}} \right)^{\frac{-\gamma}{\theta-1}} \left(R_{W,t+1}^R \right)^{\frac{\rho-\gamma}{1-\rho}}$

where,

$$m_{t+1} = \beta \left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\gamma} \left(\frac{N_{D,t+1}}{N_{D,t}} \right)^{\frac{1-\gamma}{\theta-1}} \tag{23}$$

In order to assess how the presence of the extensive margin modifies the asset pricing predictions of the standard consumption-based model let's calculate the Hansen-Jagannathan bounds of the model, when varieties are properly accounted. The Sharpe Ratio is given by the following equation:

$$\left| \frac{E\left(R^R_{E,t+1} - R_{f,t}\right)}{std\left(R^R_{E,t+1} - R_{f,t}\right)} \right| \leq \frac{std\,(m_{t+1})}{E\,(m_{t+1})}$$

which under log-normality can be expressed in the model as:

$$\frac{std\,(m_{t+1})}{E\,(m_{t+1})} = \left[\gamma^2 \sigma^2_{\Delta c_{t+1}} + \left(\frac{\gamma-1}{\theta-1} \right)^2 \sigma^2_{\Delta n_{D,t+1}} + 2\gamma \left(\frac{\gamma-1}{\theta-1} \right) \sigma_{\Delta c_{t+1} \Delta n_{D,t+1}} \right]^{\frac{1}{2}} \tag{24}$$

where $\sigma_{\Delta c_{t+1}}$ is the standard deviation of the consumption growth rate, $\sigma_{\Delta n_{D,t+1}}$ is the standard deviation of the growth rate of the number of varieties and $\sigma_{\Delta c_{t+1} \Delta n_{D,t+1}}$ is the covariance term. The model implies a higher Sharpe Ratio to the extend it can deliver a positive correlation between the number of goods and real consumption growth. If changes in the number of varieties are ignored the Sharpe Ratio reduces to: $\gamma\sigma_{\Delta c_{t+1}}$, as in the standard model.

5. Business Cycles and Asset Pricing Predictions

This section presents the quantitative results of the simulation of the benchmark model and explore its predictions regarding aggregate quantities and prices.

For aggregate quantities I assess the performance of the model using the standard real business cycle moments. I follow the usual procedure and compute correlations and volatilities of a log-linear first-order approximation of the model and compute deviations from trend using the Hodrick-Prescott filter consistent with quarterly frequencies. In order to compare the model to the data, I use the aggregate output, consumption and investment deflated for the variety effect as defined in equation (4)

For asset prices, I compute the volatility of the stochastic discount factor implied by the model starting from the standard Euler equation:

$$1 = \beta E_t \left[\left(\frac{C^R_{t+1}}{C^R_t} \right)^{-\gamma} R^R_{E,t+1} \right] = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{N_{D,t+1}}{N_{D,t}} \right)^{-\lambda} R^R_{E,t+1} \right] \tag{25}$$

where $\lambda = \gamma / (\theta - 1)$. Note that this is the data-consistent Euler equation which is different from the theoretical Euler equation of the model, as presented in equation (1). The latter is defined in units of the final good, which include the variety effect, while the former ignores the effect of varieties. From here, once can compute the excess returns of equity and the risk free rate in the model as:

$$E_t\left(r_{E,t+1}\right)-r_{F,t}=\gamma cov\left(\Delta c_{t+1},r_{E,t+1}\right)+\lambda cov\left(\Delta n_{D,t+1},r^e_{E,t+1}\right) \tag{26}$$

$$r_{F,t}=\gamma E_t\left(\Delta c^R_{t+1}\right)-\frac{1}{2}\gamma^2 V\left(\Delta c^R_{t+1}\right)-\log\beta \tag{27}$$

where, $r_{E,t+1} = log\left(R^R_{E,t+1}\right)$, $r_{F,t} = log\left(R_{f,t}\right)$, $\triangle c_{t+1} = log\left(\frac{C_{t+1}}{C_t}\right)$, $\triangle c^R_{t+1} = log\left(\frac{C^R_{t+1}}{C^R_t}\right)$ and $\triangle n_{D,t+1} = log\left(\frac{N_{D,t+1}}{N_{D,t}}\right)$.

Under the assumption that the model is the true data generating process, the volatility of the stochastic discount factor will be determined by the volatility of consumption, the volatility of the extensive margin of production and their respective correlation.

5.1. Calibration of Model Parameters

The values for the set of relevant parameters are set in order to match key features of the US economy at a quarterly frequency. A summary of the calibrated parameters of the benchmark model is presented on Table 1. Regarding preferences, I follow the standard choices in the RBC literature and set the discount factor of households (β) to 0.99 and the Frisch-Elasticity of Labor Supply (ϵ) to 2. The disutility of working parameter (A) is chosen to target a level of hours worked of 1/3 in the non-stochastic steady state. The risk-aversion parameter (γ) is set to 10, which is a low number relative to the literature regarding risk premium, but slightly higher than the usual choice for the RBC literature. I also follow the usual choice for the depreciation rate of the capital stock (δ_K) of 0.025 and set α equal to 0.4 in order to have a capital income share of 0.3.[12]

Absent a target for the non-stochastic steady state number of firms, both sunk entry costs (f_e) and per-period operating costs (f) are normalized to one. Following Bernard and Jensen (2004), the Pareto shape parameter κ is set to 3.1 in order to reproduce in the model the standard deviation of log sales of 1.67 in US plants. The support of the distribution of productivities z_{min} is set to 1. The parameter ζ_e that penalizes firm creation is set so as to match the volatility of new establishments in the data. The death parameter (δ) is set to 0.025, which is consistent with an exogenous exit rate of 10% in annual terms, as other papers with entry such as Restuccia and Rogerson (2008), Ghironi and Melitz (2005) and Bilbiie et al. (2012).

For the benchmark specification I calibrate the elasticity of substitution among domestic varieties (θ) to a value of 3.5, which implies a mark-up over marginal cost of 40%.[13] This parameter is key for the asset pricing implications of the model because it governs the degree of love for variety in the model by shaping the curvature of the price deflator relative to the number or firms. Moreover, a value close to one can potential deliver an arbitrary large risk premium. In the sensitivity analysis I discuss the predictions of the model under two alternative values for the elasticity of substitution: 2.5 and 5.

Finally, the calibration takes into account that the model delivers a endogenous TFP series that may not coincide with the estimated Solow residuals as measured in the data. Aggregate output in the model is a function of the amount of productive labor, rather than total working hours

[12] In the model α is not exactly the capital income share given the presence of profits

[13] Given that firms pay sunk entry and fixed exporting costs, the mark-up over average costs is lower

Table 1: Parameter Values Baseline and Sensitivity

	Parameters and Values	Targets / Robustness
Baseline		
Households	$\beta = 0.99, \gamma = 10, \epsilon = 2, A = 5$	4% Interest Rates, 1/3 Hours Worked
Production	$\alpha = 0.4, \delta_K = 0.025$	Capital Income Share, Capital-Output Ratio
Distribution of Firms	$\kappa = 3.1, z_{min} = 1, \delta = 0.025$	Log Sales US Plants, Exit Rate
Elasticity of Substitution	$\theta = 3.5$	Mark-up over marginal cost
Sunk and Fixed Costs	$f_e = 1, f = 1, \zeta_e = 2$	Volatility Entry
Stochastic Process	$\rho_z = 0.91, \sigma_v = 0.019$	$\rho_{TFP} = 0.98, \sigma_\psi = 0.07$
Sensitivity Analysis		
Lower Elasticity (I)	$\theta = 2.5 , \zeta_e = 3.8$	Higher Mark-ups
Higher Elasticity (II)	$\theta = 5 , \zeta_e = 1.8$	Lower Mark-ups
No Extensive Margin (V)	$\zeta_e = 100,000, f = 0 , \bar{z}_D = 1$	Fixed Number of Firms
No TFP Adjustment (VI)	$\rho_z = 0.97, \sigma_v = 0.007$	No TFP Adjustment

Notes: This table reports the parameter values of the baseline model at a quarterly frequency and the main targets. See the main text for a full description of the calibration. It also reports the parameter values used for the sensitivity analysis.

that also include entry and fixed costs of operation, which are labor intensive. However, when estimating Solow residuals from the data there is no such distinction.

The wedge between the model's TFP and the exogenous productivity process can be better seen from the expression that defines the former:

$$TFP = N_D^{\frac{\alpha}{\theta-1}} Z \frac{L_p^{1-\alpha}}{L^{1-\alpha}} \tag{28}$$

Cyclical changes in the number of firms and in the labor share devoted to production appear as changes in the model's TFP. In the standard model, the number of firms is constant and all labor is devoted to production, so both TFP and Z coincide. In order to make the model-based TFP consistent with the Solow residuals estimates, the stochastic process for exogenous aggregate productivity, Z_t, has to be defined such that when fed into the model, it delivers a TFP process with the same properties estimated from the data.

The standard RBC calibration for TFP, despite featuring some variation, exhibits fairly persistent processes and innovations with a standard deviation close to 1%. For this exercise, I follow the Solow residual's estimates of King and Rebelo (1999) which implies the following TFP calibration target:

$$TFP_t = 0.98 \cdot TFP_{t-1} + \psi_t \tag{29}$$

with a standard deviation σ_ψ of .007.

The exogenous stochastic process, Z_t, is defined by following expression:

$$Z_t = \rho_z Z_{t-1} + v_t$$

I start with a guess for $\{\rho_z, \sigma_v\}$, simulate the model for a large number of periods, and then estimate an autoregressive process for the simulated TFP series, in the same way the data is treated.[14] I iterate this procedure until the model based estimation of the TFP processes matches

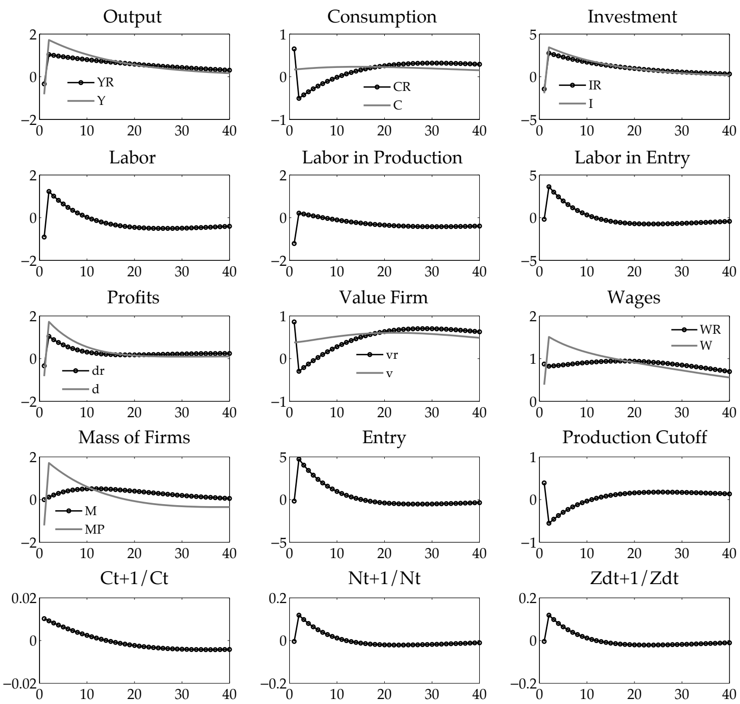
[14] Solow residuals in the data are expressed as percentage deviations from a linear trend. I apply the same definition and use a linear trend for this exercise

exactly the autocorrelation and the standard deviation of the shocks. The resulting vector of estimates for Z_t that make the model's TFP match the data in the benchmark specification is $\{\rho_z = 0.9, \sigma_v = 0.017\}$. Note that the persistence parameter of the exogenous shock is lower and the standard deviation is higher than those for the TFP process. This is explained, as I will discuss in the next section, by the immediate fall in the labor share of production as the economy reallocates labor into firm creation and the subsequent increase in the number of firms. In other words, firm entry allows the model economy to smooth over time the productivity shock.

5.2. Benchmark Model

Figure 1 presents the impulse-response functions of the main variables of the model after a positive productivity shock to aggregate productivity.

Figure 1: Impulse-Responses



Notes:This figure shows the impulse response functions of the main variables of the model in % deviations of steady-state after one standard deviation positive productivity shock. The thin lines correspond to the variables in units of the final good (welfare-based), while the thick lines correspond to the real variables as defined by equation (20) which are comparable to the data.

A positive productivity shock induces higher output, consumption and investment. It also translates into a large number of productive firms as the economy increases labor devoted to entry of new establishments. At the time of the shock, labor devoted to firm entry increases at expense of a decline in productive labor. This fall in productive labor translates into a short-lived fall in output and in investment that unwinds as labor shifts-back to production and new firms become operative. There is also a temporary reduction in the number of available varieties associated to an increase in the production cut off as wages rise as a result of greater labor demand. Firm profits fall at the time of the shock because of the temporary increase in the production cut off, but turn positive afterwards, inducing entry of new firms. Equity returns follow a similar pattern. All together, a positive productivity shock leads to higher economy activity, profits and returns and to an expansion in the number of firms.

The increase in the number of firms manifests in wider gap between variables denominated in units of the final good and real variables corrected to exclude the variety effect present in the model. The first row of Figure 1 presents the response of output, consumption and investment both in terms of units of the final good and in real terms. Note that real variables, which are comparable to the data, differ to nominal variables to the extent that the number of firms in the economy change. For the case of output and investment, real variables tend to show a more muted response than their nominal counterparts because of the positive correlation between the mass of firms and the intensive margin. On the contrary, real consumption is more volatile than consumption in units of the final good because households optimally smooth consumption of the final good and take investment and entry decisions accordingly. The higher implied volatility of real consumption allows the model to improve its asset pricing implications relatively to the standard RBC model that neglects any changes in the number of consumed varieties.

Table 2 summarizes statistics for the second moments of real quantities in the data and in the model. As mentioned before, the statistics for real quantities are computed using the Hodrick-Prescott Filter, while the growth rate of consumption and the number of varieties are taken directly from the model's simulated moments using a second-order approximation.[15]

The benchmark specification can account reasonable well for the second moments of aggregate quantities and match the Sharpe Ratio observed in the data. The model over-predicts the volatility of labor because of the additional volatility associated to the labor devoted to firm creation, but fits well the volatility and correlation of other aggregate variables with the exception of the volatility of investment that is larger than in the data. The equity premium in the model is around 1% in annual terms, and the risk free-rate is slightly higher than the one observed in the data. Although the benchmark calibration cannot fully account for the excess returns of stocks as observed in the data, it delivers much more realistic predictions than the standard model.

5.3. Sensitivity Analysis

Columns (I) though (IV) of Table 3 present the results to various calibration and modeling changes. As it can be seen from Columns (I) and (II), different values for the elasticity of substitution have an important effect on the statistics for both quantities and asset prices. A lower elasticity of substitution allows the model to over-predict the risk premium by generating a too large volatility of real consumption growth. Regarding quantities, the volatility of output under this specification is lower and the relative volatility of labor and investment relative to output are counterfactually excessive. This specification generates enough risk premium at the expense of performing badly in terms volatility of output, labor and investment and over-shooting the

[15]The data for real quantities is at quarterly basis and Hodrick-Prescott filtered with a smooth parameter of 1,600. The only exception is the relative volatility for the number of firms which is computed using annual time series of output and the BDS data using the 1978-2011 period. For this annual statistic I use a smooth parameter of 6.5 as suggested by Ravn and Uhlig (2002)

Table 2: Quarterly RBC Statistics and Asset Prices

	Data	Model				
		Base	Lower Elas (I)	Higher Elas (II)	No Entry Margin (III)	No TFP Adj. (IV)
Volatility Aggregate Quantities (First-Order, HP filtered)						
σ_{Y^R}	1.81	1.90	1.53	2.25	3.09	0.61
$\sigma_{C^R}/\sigma_{Y^R}$	0.74	0.73	3.24	0.36	0.05	0.94
$\sigma_{I^R}/\sigma_{Y^R}$	3.00	7.85	23.46	5.43	3.87	6.15
σ_L/σ_{Y^R}	0.99	1.23	2.42	1.02	0.36	1.68
$\sigma_{L_p}/\sigma_{Y^R}$	na	1.17	3.48	0.79	0.66	1.67
σ_N/σ_{Y^R}	0.71	0.32	0.35	0.31	0.00	0.22
σ_{Ne}/σ_{Y^R}	4.00	4.00	4.00	4.00	0.00	3.53
Domestic Correlations (First-Order, HP filtered)						
$corr_{Y,C}$	0.86	0.48	0.29	0.58	0.56	0.15
$corr_{Y,I}$	0.95	0.99	0.99	0.99	1.00	0.99
$corr_{Y,L}$	0.87	0.93	0.91	0.94	0.78	0.78
$corr_{Y,N}$	0.51	0.58	0.51	0.58	0.00	0.52
$corr_{Y,N_e}$	0.16	0.83	0.61	0.89	0.00	0.94
Financial Moments (Second Order Simulations and lognormal pricing formulae)						
$E(R_f)$	0.84	1.50	3.43	3.19	3.43	3.30
$E\left(R_E^R - R_f\right)$	6.18	1.04	6.39	0.37	0.0007	0.15
$std\,(m)\,/E\,(m)$	0.34	0.33	0.81	0.20	0.026	0.12

Notes: This table compares the quarterly RBC statistics and the risk premium of the data, the baseline model and different specifications as explained in the Sensitivity Analysis section. The first block reports the volatility of real output (σ_Y) and the volatility of real consumption (σ_C), real investment (σ_I), labor (σ_L), productive labor (σ_{L_p}), the number of firms (σ_N) and firm entry (σ_{Ne}), relative to real output. The second block presents the correlation of the main variables with respect to real output. The last block reports asset pricing moments from the data and the model. The Sharpe Ratio, the equity premium and the risk-free rate of the model are computed using equations (25), (26) and (27), respectively. Details on the data sources are available on the Appendix.

volatility of real consumption. On the contrary, the specification with a much larger value for the elasticity of substitution captures well the dynamics of aggregate quantities but cannot generate enough volatility of real consumption and risk premium. This is consistent with the fact that the model convergences to a model in which the variety effect plays no role when the elasticity of substitution is large enough.

Column (III) examines the role of the extensive margin for the dynamics of the model. Without any changes in the number of goods, the asset pricing implication of the model are equal to the standard RBC as the model fails to generate enough movement in the stochastic discount factor. The volatility of the consumption growth rate is too low and with it the implied risk-premium. Moreover, the real output volatility is too large while the relative volatilities of consumption, investment and labor are counterfactually small. The excessive volatility of real output is mainly result of the re-calibration exercise of the exogenous productivity process.

Finally, I discuss the predictions of the model if the calibration of the exogenous productivity process ignores the fact that the model's TFP displays different stochastic properties. In this case the model exhibits too little volatility of real output, consumption and all other aggregate variables. Accordingly, the model predicts a lower risk premium. This specification shows that adjusting the TFP process is relatively more important for the fit of real quantities than for the asset pricing predictions.

6. Pricing Errors in the Consumption-Variety Asset Pricing Model

This section estimates the cross-sectional pricing errors of the standard consumption-based asset pricing model (CCAPM) augmented to include a measure related to the growth rate of the number of varieties, which for convenience I label here the Consumption-Variety Asset Pricing Model (CVAPM).

There is a large literature devoted to study the empirical failures of the CCAPM at explaining cross-sectional asset pricing data. The purpose of this section is to test whether considering the extensive margin of production has any implications for the empirical fit of the standard CCAPM as it has for the theoretical asset pricing predictions of the model as discussed in the previous section.

Ideally, one would like to add to the CCAPM a measure of the growth rate of the number of products in the whole economy to test its asset pricing predictions. Lacking a comprehensive and long enough time series for the number of goods, the exercise here presented is restricted to test the pricing errors of the CVAPM model when growth variety is proxied by alternative measures. As before, I use data on establishment entry. In particular, I calculate the growth rate in the number of establishments, the entry rate of establishments and the share of jobs added to the economy by new establishments as proxy measures of changes in the set of products.[16]

Additionally to these measures, I also use the growth rate of the number of firms and patent applications in the US as proxies. Arguably these two measures are less adequate to test the asset pricing implications of the model. Firm data is a coarser measure than the establishment data as many firms in the economy are multi-unit. Furthermore, firm entry could be related to other factors of market risk not present in the model and study elsewhere such as financing constraints or growth opportunities. On its turn, patent applications are an imperfect measure of growth variety, as many patent applications produce no new goods. One advantage of using either of these two variables is that they are available for a much longer sample, so the fit of the CVAM can be tested with more degrees of freedom.

6.1. Establishment, Firm and Patents data

The data for the number of establishments is from the Longitudinal Business Database (LBD) of the Business Dynamic Statistics (BDS) from 1977 to 2011.[17]The Business Dynamic Statistics defines an establishment as a fixed physical location where economic activity occurs. A firm may have one establishment (a single–unit establishment) or many establishments (a multi–unit

[16]As new varieties are introduced in the economy by new or existing establishments, the number of establishment is the lower bound of the number of goods. There is evidence in the manufacturing sector that product innovation is more volatile than establishment creation. Using data from the quinquennial Census of Manufacturers, Bernard et al. (2010) document that product innovation by existing firms, either single or multi-unit, is highly volatile and pro-cyclical, which suggests that the CVAPM would exhibit even more volatility in the household pricing kernel and, perhaps, could have a better empirical asset pricing fit, if it incorporates a direct measure of product innovation.

[17]http://www.census.gov/ces/dataproducts/bds/

Table 3: Descriptive Statistics

Variable	Mean (%)	SD (%)	Correlations: Stock Market Returns	Correlations: Consumption
Period 1978:2011				
Stock Market Excess Returns	7.06	16.39	1	
Consumption	2.59	1.58	0.11	1
Establishment Growth	1.02	1.37	0.18	0.45
Establishment Entry Rate	0.12	0.016	0.24	0.25
Job Creation New Establishments	0.07	0.008	0.18	0.44
Firm Growth	1.12	1.54	0.15	0.45
Patents	4.91	4.63	0.06	0.25
Period 1930:2011				
Stock Market Excess Returns	7.38	20.20	1	
Consumption	2.98	2.77	0.24	1
Firm Growth	1.16	2.62	0.02	0.07
Patents	2.43	7.79	0.12	0.40

Notes: This table reports the mean, standard deviation and correlation of excess market returns, the growth rate of consumption, the growth rate of the number of establishments, the establishment entry rate and the job creation of new establishments as percentage of total employment, the growth rate of the number of firms and the growth rate of patent applications. See Appendix and the text for the details on the sources and the definition of the different variables.

firm). Establishments are primarily non-public as the LBD excludes governmental establishments except for liquor stores (SIC 592), wholesale liquor establishments (SIC 518), depository institutions (SIC 60), federal and federally sponsored credit agencies (SIC 611), and hospitals (SIC 806). From this data I calculate the growth rate of the number of establishments in the US economy (*establishment growth*), the entry rate of new establishments define as the ratio of birth units relative to the total number of establishments (*establishment entry rate*) and the job creation of birth establishments relative to total employment (*job creation new establishments*). The birth of new units is defined based on the year an establishment first reports positive employment in the LBD. Since the LBD is based on administrative rather than sample data, sampling error is not likely to play an important role in the quality of these indicators.

The data for the number of firms (*firm growth*) in the US economy comes from two sources: the Economic Report of the President and the Business Dynamic Statistics. From 1932 to 1964, the number of firms is from the table on Business Population and Business Failures 1932-64, Table B-72 of the Economic Report of the President.[18]The series on Operating Business was discontinued in 1965, so for the following years information on New Business Incorporations is taken from the same report until 1976. From 1977 to 2011 the data is from the Business Dynamics Statistics. Finally, the time series for patents applications (*patents*) is collected and reported by the US Patent and Trademark Office and it is available for the whole period of the estimation, namely 1932 to 2011.[19]The growth rate of total patent applications includes utility,

[18]http://fraser.stlouisfed.org/publication/?pid=45
[19]http://www.uspto.gov/web/offices/ac/ido/oeip/taf/reports.htm

design and plant patents.

Table 3 presents the basic statistics regarding the excess return of the stock market, the growth rate of consumption, the growth rate of establishments (*establishment growth*), the entry rate of new establishments (*establishment entry rate*), job creation of birth establishments (*job creation new establishments*), the growth rate of the number of firms (*firm growth*) and the growth rate for patent applications in the US (*patents*). Because the establishment data is only available from 1978, I present the main statistics of all variables for the most recent period and the statistics of the data on firms and patents for the longer sample (1932:2011).

All the series related to establishment growth are positively correlated to the growth rate of consumption and with the excess of equity returns. This is consistent with previous works that document that establishment entry is pro-cyclical (Lee and Mukoyama (2012), Bilbiie et al. (2012) and Fattal Jaef and Lopez (2014)). The growth rate in the number of firms and patent applications are also pro-cyclical and positively correlated with equity returns, in particular during the 1978-2011 period for the former and in the longer sample for the latter.

A simple inspection of these statistics suggest that augmenting the standard model to include any of these proxies for consumption growth at the extensive margin should increase the volatility and predicting power of the model's pricing kernel. In order to formalize this idea I test whether the CVAPM delivers lower pricing errors than the CCAPM for the cross-sectional returns of selected equity portfolios. First, I estimate from a set of assets, formed by the Fama and French's 30 industry portfolios ($R_{t+1}^{E,i}$), the betas from the time-series regressions for each asset i:

$$R_t^{E,i} = a_i + \beta_i^{'} f_t + \varepsilon_t^i \tag{30}$$

where $\beta_i = cov\left(f_t, R_t^{E,i}\right)/var\left(f_t\right)$. Then, I estimate the factor risk premia λ from a cross-asset regression on average excess returns:

$$E_T\left[R_{t+1}^{E,i}\right] = \beta_i \lambda \tag{31}$$

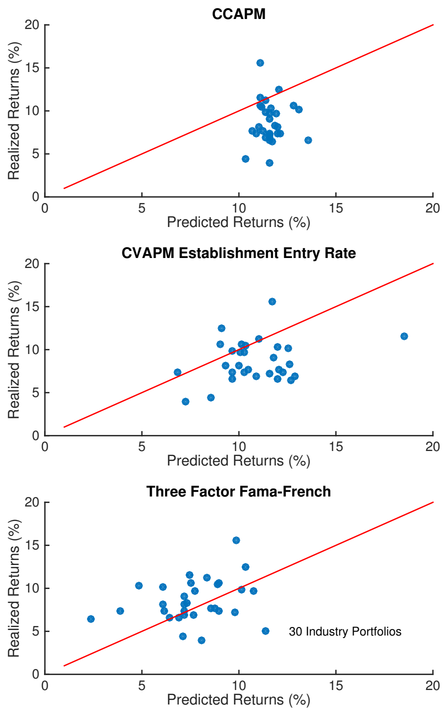
For the standard CCAPM, the factor f_t is the growth rate of real non-durable consumption. For the CVAPM, I add one at a time any of the proxies of growth variety from the establishment data. Alternatively, I also test the growth rate of firms and patent applications as factors. I compare the results with the well known Fama-French three-factor model, which includes as factors the excess return of the market (R_t^m), the difference between returns on small and large stocks (small minus big -*smb*), and the difference between returns on high and low book-to-equity market ratio (high minus low- *hml*).[20]

Figure 2 presents the realized returns of the Fama and French's 30 industry portfolios for the 1978 to 2011 period and the predicted returns for the CAPM, the CVAPM using the establishment entry rate and the three-factor Fama French model. This figure gives a visual impression of how the CVAPM improves the cross-sectional fit of the CAPM and performs better than the three-factor model despite having a lower number of factors.

Table 4 presents the results of the estimation of the different factor models for two different sample periods: 1978 to 2011 and 1932 to 2011. The the OLS estimates of the $\lambda's$ of each specification with their corresponding t-statics and the Shanken (1992) t-statistic so as to correct for

[20]The three Fama-French factors - market excess returns, small minus big (*smb*), and high minus low (*hml*)- and the 30 industry portfolios at the four-digit SIC code are all taken from the Data Library of Kenneth French:http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Figure 2: Realized and Predicted Returns for the Fama and French's 30 industry portfolios from 1978 to 2011



Notes: The graph plots the observed and fitted excess returns according to the CAPM, the CVAPM using the establishment entry rate and the Fama-French three-factor model. Pricing errors are generated using a gls estimation of equation 31

sampling errors that arise because the betas are estimated in a first-stage time-series regression. The table also reports the R^2 of the GLS estimation of the cross-sectional regression as suggested by Lewellen et al. (2010), and the χ^2 test for the hypothesis that the Shanken-corrected GLS pricing errors on the portfolios are jointly zero.

These statistics show that the CVAPM model delivers lower pricing errors than the CCAPM and fits better the cross-sectional regression. The various specifications of the CVAPM model have higher R^2 and lower levels for the χ^2 test. Relative to the three factor model, the CVAPM is able to explain better the cross-sectional returns of the industry portfolios for the shorter sample. The empirical fit of the CVAPM is far from ideal as it delivers factor coefficients with relative low t-statistics and, for some specifications, with the wrong sign. Note that the *smb* and *hml* factors of the Fama-French model also exhibit low t-statistics, most likely as a result of the short sample period of the estimation exercise.

The improvement of the pricing fit of the CVAMP relative to the CAPM and the fact that it delivers lower errors than the three-factor model when pricing industry portfolios, raises the bar of the failure of the standard consumption model and suggests that the extensive margin of consumption plays an important in shaping the macroeconomic risk to which households are

Table 4: Pricing Errors Cross-Sectional Regressions CCAPM, CVAPM and 3-Factor Model

Sample 1978-2011			R^2_{GLS}	χ^2
CCAPM				
Consumption				
0.80 [0.38] (0.22)			0.07	16.43
CVAPM				
Consumption	Establishment Growth			
0.52 [0.27] (0.15)	-0.36 [-0.12] (-0.11)		0.15	17.11
	Establishment Entry Rate			
1.20 [0.47] (0.30)	0.02 [0.63] (0.20)		0.86	15.69
	Job Creation New Establishments			
1.14 [0.45] (0.29)	0.01 [0.55] (0.21)		0.63	16.09
	Firm Growth			
0.29 [0.15] (0.08)	-0.97 [-0.27] (-0.24)		0.52	15.70
	Patents			
1.24 [0.52] (0.29)	-3.45 [-0.28] (-0.24)		0.73	12.01
3-Factor (Fama-French)				
Excess Returns	Small minus Big	High minus Low		
9.45 [2.69] (0.51)	-3.43 [-0.34] (-0.22)	-1.15 [-0.06] (0.03)	0.30	15.61
Sample 1932-2011			R^2_{GLS}	χ^2
CCAPM				
Consumption				
0.80 [0.38] (0.22)			0.07	16.43
CVAPM				
Consumption	Firm Growth			
3.04 [0.46] (0.39)	3.10 [0.37] (0.34)		0.18	1.34
	Patents			
4.09 [0.42] (0.33)	1.74 [0.12] (0.10)		0.17	1.23
3-Factor (Fama-French)				
Excess Returns	Small minus Big	High minus Low		
9.44 [2.74] (0.43)	0.10 [0.01] (0.01)	1.24 [0.07] (0.05)	0.21	0.84

Notes:The table reports the OLS estimates of the factor loadings, t-statistics in brackets, Shanken t-statistics in parentheses, the R^2 of the GLS estimation of equation 31, and the χ^2 test of the null hypothesis that all Shanken-corrected GLS pricing errors are joinlty zero for the CAPM, the CVAPM using different proxies for growth variety and the Fama-French three-factor model.

exposed and from which asset prices are endogenously derived.

7. Conclusions

The findings of this paper suggest that changes in the number of goods along the cycle are important for households' welfare and their exposure to aggregate risk. Most of the literature has ignored this effect by working with models with a fixed set of goods. This papers shows, empirically and theoretically, that the extensive margin of production helps to deliver larger volatility in the stochastic discount factor of households. Most of the asset pricing literature has taken the route of revisiting the standard model by assuming more complex preferences that have richer predictions regarding agent's risk aversion. This paper suggests a complementarity path by exploring the unmeasured welfare effects of cyclical changes in the set of goods. The findings of this paper suggest the need for further research into the effect of changes alone the extensive margin and the degree to which households are exposed to fluctuations in economic activity through these unexplored channel.

In this paper I focus on the implications of the extensive margin in the context of a closed-economy. Unmeasured welfare changes in varieties across countries could be an important aspect of pricing of exchange rates and other asset prices arising from international trade risk. This international dimension is out of the reach of this paper, but it could be a promising direction to explain some of the puzzling features of pricing aggregate risk across countries.[21]

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[21] Scanlon (2009)and Gavazzoni and Santacreu (2014) have recent work on this direction

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Appendix

Data Appendix

The data for output, investment and consumption is from the NIPA accounts. Consumption refers to consumption of non-durable and services. Labor is total hours in the economy from Cociuba et al. (2012). Statistics for real quantities are computed using the Hodrick-Prescott Filter with a $\lambda = 1600$. Financial data regarding nominal short term interest rates, (3-Month T-Bills) and Inflation (Annual CPI for All Urban Consumers, CPIAUCSL 1982-84=100) is from the Fred St. Louis Data[22]. The return on equity is measured as the total return on investment in the S&P 500. The three Fama-French factors and the 30 industry portfolios at the four-digit SIC code are all taken from the Data Library of Kenneth French

Measurements of Variety Growth

For the 1978 to 2011 period the measurements of variety growth proxy by firm and establishment entry are from the Longitudinal Business Database of the Business Dynamic Statistics. The growth rate of firms and entry are at the national level. The data for the number of firms (*firm growth*) in the US economy comes from two sources: the Economic Report of the President and the Business Dynamic Statistics. From 1932 to 1964, the number of firms is from the table on Business Population and Business Failures 1932-64, Table B-72 of the Economic Report of the President. The time series for patents applications (*patents*) is from the US Patent and Trademark Office.

[22] http://research.stlouisfed.org/fred2/