Financial Shocks, Intangible Capital and
the Cyclical Behavior of Unemployment∗

Jose Ignacio Lopez Virginia Olivella
HEC Paris † Banque de France ‡

March 8, 2016

Abstract

We study the effects of financial shocks on labor markets in a model with both labor and financial frictions, two types of productive capital, physical and intangible, and in which only the former serves as collateral. A tighter borrowing constraint in this environment leads to a fall in credit and investment, skewed in detriment of intangibles, which in its turn lowers the marginal product of labor and reduces the incentives to hire workers. When feeding into the model financial shocks estimated from the data, we find that they explain labor outcomes during the last three downturns in the US, including the sharp increase in unemployment during the great recession.

Keywords: Financial Shocks, Intangible Assets, Business Cycles, Employment Volatility
JEL Codes: E24, E32, E44

1 Introduction

The last financial and economic crisis in the US has brought renewed interest in models of financial frictions and their impact on economic fluctuations.1 Whereas most of the literature prior to the

†Corresponding author. 1 Rue de la Libération 78351 Jouy en Josas Cedex, France. Email: lopez@hec.fr. Tel: +33 621526951
‡1 Rue de la Vrillère 75001 Paris, France. Email: Virginia.OlivellaMoppett@banque-france.fr. The views expressed in this paper are those of the authors alone and do not reflect those of the Banque de France.
1See Quadrini (2011) for a survey on the literature
great recession focused on the role of the financial sector in propagating real shocks, new research has began to explore the importance of financial shocks, that is, shocks that originate in the financial sector and directly affect the ability of firms to raise external funds.\(^2\)

In this paper we investigate the impact of financial shocks on the real economy, particularly on the business cycle dynamics of labor markets. We do this in the context of a model with financial and labor market frictions and in which asset tangibility plays an important role in the financing conditions of firms. So far the literature studying financial shocks has done so by discussing models in which the assets used as collateral to finance production are either non-reproducible or equal to the whole capital stock. These assumptions, although useful for highlighting specific mechanisms in which financial shocks affect the economy, are not completely satisfactory. The amount of pledgeable assets in the economy fluctuates over the business cycle as the stock of physical capital accepted as collateral, such as structures and equipment, changes with firms’ investment decisions. Furthermore, part of the capital stock used in production, such as intangible capital, cannot be pledged as collateral given its inalienable nature. We argue in this paper that financial shocks affect the real economy not only by changing the total amount of collateral and credit available for firm financing, but also by altering the optimal mix between pledgeable and non-pledgeable assets in the economy.

We start by documenting new facts about the relationship between asset tangibility, external financing and employment. We construct measures of tangible and intangible capital at the firm level from 1980 to 2010 using accounting information on U.S. public firms from Compustat data. Aggregating over the firm level measures, we find that the ratio of intangible-to-tangible assets for the whole economy declined during the great recession, suggesting that the financial crisis had a stronger adverse effect on investment in non-pledgeable assets. Using panel fixed effect regressions, we also find that companies with higher shares of intangible assets have lower leverage ratios on average and had to de-leverage more during the financial crisis than firms with lower intangible-to-tangible ratios. Finally, we document that firms with higher shares of intangible assets experienced larger declines in employment during the last recession.

Motivated by these findings, we build a model with financial frictions, in the form of a collateral constraint, and in which entrepreneurs invest on tangible and intangible capital, but cannot use the latter as collateral because it has no residual value for creditors.\(^3\) Our economy is populated by relative impatient entrepreneurs and a representative household that supplies labor and provides funds to finance the entrepreneurs’ productive activity. Entrepreneurs run a constant returns to scale production function using labor and two types of capital: tangibles and intangibles. A fraction of the labor input is used for recruiting activities and each period there is an exogenous

\(^2\)See, for example, Christiano, Motto, and Rostagno (2010), Jermann and Quadrini (2012), Kiyotaki and Moore (2012), Gilchrist and Zakrajsek (2012) and Khan and Thomas (2013), for models with financial shocks.

\(^3\)The introduction of collateralized debt as a way of modeling pro-cyclical credit supply follows Kiyotaki and Moore (1997).
and constant separation rate between workers and firms. Because of limited enforceability of contracts, entrepreneurs can only borrow up to a fraction of the value of their stock of tangible assets. Entrepreneurs also need working capital, which introduces a labor wedge in the economy.

We find that a financial shock, modeled as a tightening of the borrowing constraint, can generate significant volatility in the unemployment rate and the labor market tightness as well as in the overall level of employment. When feeding into the model productivity and financial shocks estimated from the data, we find that financial shocks are capable of replicating the behavior of labor market variables during the last three economic crisis in the US, including the sharp increase of the unemployment rate during the great recession. In line with the existing literature, we also find that productivity shocks fail to produce enough movements in labor market variables.\footnote{Since the seminal work of Merz (1995), Andolfatto (1996) and the influential paper by Shimer (2005), there has been a lot of work devoted to study the quantitative performance of the search and matching Mortensen and Pissarides (1994) model subject to productivity shocks. Hornstein, Krusell, and Violante (2005) and Shimer (2010) present a summary of this literature.}

The high volatility of employment in the model generated by financial shocks can be explained as follows. A tighter borrowing constraint forces entrepreneurs to reduce their debt and scale down their operation. It also gives them incentives to reallocate resources towards the pledgeable asset at the expense of investment in the non-collateralized one, so as to offset the tightening in debt financing conditions. The overall effect is a decline in aggregate capital driven by the decline in intangible investment that reduces the marginal product of labor and thus the incentives to hire workers. Additionally, as a result of the need for working capital, the tightening of the borrowing constraint generates an increase in the shadow cost of financing labor, providing firms with further incentives to cut back on employment. With respect to TFP shocks, the results in Shimer (2005, 2010) carry over to our model with financial frictions. Productivity shocks cannot on their own produce enough volatility of employment because wage dynamics offset any changes in labor productivity.

The assumptions that only tangible assets can be pledged as collateral and that entrepreneurs need working capital are quantitatively important for the main results of the model. In particular, when we consider a model in which there is only one type of capital that can be entirely used as collateral, we find that a negative financial shock has a hard time generating a recession and a significant fall in employment. Absent the reallocation effect between pledgeable and non pledgeable assets present in the benchmark specification, a model with only physical capital exhibits a smaller reduction in the capital stock when a financial shock hits the economy given that, as in the benchmark model, the entrepreneur has incentives to accumulate collateral to counteract the tightening of the borrowing constraint. Thus a financial shock in this environment leads to a smaller decline in the marginal product of labor, employment and output. A model without working capital also reduces the impact of a financial shock on employment, such that employment volatility is 85% that of the benchmark model.
This paper is related to three different strands of literature. The first one studies models with financial and labor market frictions such as Wasmer and Weil (2004), Caggese and Cuñat (2013) and Petrosky-Nadeau and Wasmer (2013). This paper differs from theirs as we consider shocks that directly affect the financing conditions in the economy. These shocks resemble the financial shocks studied in Jermann and Quadrini (2012), in which the financial sector acts too as a source of business cycles. Different from their work that highlights the interaction between financial frictions and the firm’s equity and debt flows in a model with frictionless labor markets, we focus on how financial shocks impact investment and hiring decisions by altering the composition of productive assets. Monacelli, Quadrini, and Trigari (2011) also study how labor markets respond to shocks on financing conditions. Our paper differs from theirs in that in our model the financial shock is transmitted through the standard credit channel (higher cost of financing employment), while in theirs financing costs are constant over time. In their environment a reduction in borrowing puts firms in a less favorable bargaining position with workers, which explains why after a contraction in credit their model predicts high wages and depresses job creation. Other papers that study financial shocks but in the context of models with heterogeneous agents include Guerrieri and Lorenzoni (2011), Khan and Thomas (2013), Gilchrist, Sim, and Zakrajšek (2014), Buera and Moll (2015), Bassetto, Cagetti, and De Nardi (2015) and Buera, Jae, and Shin (2015).

The second strand of literature focuses on intangible capital. Corrado, Hulten, and Sichel (2005, 2009) and more recently Corrado and Hulten (2010) have documented a secular increase in aggregate investment of intangible capital in the US since the 1950s. Hall (2001) and McGrattan et al. (2010) use the stock market and aggregate accounting data to infer from a standard investment model the dynamics of intangible capital in the US. Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2013) study the role of organization capital for the life-cycle investment and the equity returns of firms. Falato, Kadyrzhana, and Sim (2013) study external financing and cash holdings in a model with financial frictions and investment in intangible assets and Chen (2014) analyzes the effects of asset intangibility on firm dynamics in the presence of endogenous financial constraints. Both of these papers abstract from business cycle considerations. Finally, Gourio and Rudanko (2014) highlight the importance of introducing intangible capital in real business cycle models when intangibles capital is a factor of production. We contribute to this literature by documenting new empirical facts relating asset tangibility, external financing and employment during the last crisis and by building a model to study the impact of financial shocks on the firms’ decision to invest in tangible and intangible assets over the business cycles and its consequences for the labor market.

Finally, this paper relates to the literature devoted to study cyclical changes in labor market variables in models with search frictions in labor markets. We differ from this literature as we analyze financial shocks in addition to the standard productivity shocks.

The rest of the paper is organized as follows. In the next section we present evidence of the
importance of asset tangibility for external financing and employment at the aggregate and firm level. Section 3 describes our general equilibrium model with financial and labor market frictions and intangible capital. In section 4 we analyze the dynamics of the model induced by shocks estimated from the data and compute the impulse response functions of the model as well as the standard business cycle statistics. In section 5 we perform sensitivity analyses to assess the impact that changes in key elements of the model have on our results. Finally, we conclude.

2 Intangible Assets and Financial Frictions

In this section we study the relationship between asset tangibility, external financing and employment dynamics during the last two decades with a particular interest in the 2008-2009 crisis, where the turmoil in financial markets had an important effect on credit.

2.1 Measuring Intangible and Tangible Assets

We begin by constructing measures of tangible and intangible assets for public firms in the US using compustat data. The sample period goes from 1980 to 2010 and includes 20,244 firms for a total of 207,545 firm-year observations. We exclude firm-year observations reporting negative sales (Compustat item Sale), assets (item AT) or physical assets (item PPEGT) and discard firms in the regulated utility or financial sector with a primary Standard Industry Classification (SIC) code between 4900 and 4999 or between 6000 and 6999.

We use the perpetual inventory method to construct a measure of intangible capital for each firm \( i \) using the following law of motion:

\[
K_{I,i,t} = \begin{cases} 
(1 - \delta_I)K_{I,i,t-1} + XSGA_{i,t} & \text{if } t > 0 \\
INTA_{i,t} & \text{if } t = 0
\end{cases}
\]  

(1)

For the initial stock of intangible assets we use the item INTA which measures the value of reported assets that have no physical existence but represent rights to the firm. This item includes blueprints, software and patent costs, contract rights, copyrights, design costs, franchises, licenses, and organizational costs, among others. To the initial value reported in the INTA item, we add expenses on advertising, engineering, marketing and other general expenses reported in the item XGSA. The item XGSA includes most of the expenditure related to the intangible capital of the firm such as advertising, marketing, company-sponsored research, employee training, payment to internal systems and strategy consultants, distribution and internet costs.\(^5\)

\(^5\)Other papers in the literature use the same methodology to construct firm level measures of intangible capital. See Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2013), Falato, Kadyrzhanova, and Sim (2013) and Chen (2014).
this type of expenses is equal to 20% in annual terms, as in Eisfeldt and Papanikolaou (2013).\(^6\)

We assume a similar law of motion for tangible assets:

\[
K_{T,i,t} = \begin{cases} 
(1 - \delta_T)K_{T,i,t-1} + \text{CAPX}_{i,t} & \text{if } t > 0 \\
\text{PPEGT}_{i,t} & \text{if } t = 0
\end{cases}
\] (2)

The initial stock of tangible assets is taken from the PPEGT item that measures the value of all the property, plants and equipment owned by a firm. As public firms in the US have incentives to overreport the value of their physical assets because of the tax treatment of depreciation, we construct the series of tangible assets by adding to the initial stock of capital the capital expenditures reported in the cash flows statements (item CAPX) and assuming an annual depreciation rate of 8%.

2.2 Aggregate Measure of Intangible Capital

Figure 1 reports the aggregate ratio of intangible to tangible assets \(\frac{\sum_i K_{I,i,t}}{\sum_i K_{T,i,t}}\) and total employment in the US economy. Previous findings by Corrado, Hulten, and Sichel (2009); Corrado and Hulten (2010) using aggregate data and Falato, Kadyrzhanova, and Sim (2013) using firm-level data, point to a secular increase in the share of investments in intangible capital in the US in the last few decades as a result of structural changes in the production function and the organization of firms. In line with these findings, Figure 1 shows a persistent increase of the aggregate ratio of intangibles to tangible assets during the period 1980-2010.

Figure 1 also shows a decline in the share of intangible assets during the 2008-2009 crisis. The ratio of intangible-to-tangible assets fell to 0.97 in 2009 from 1.01 in 2007. This is consistent with the argument that intangible investment is more susceptible to the worsening in the financing conditions of firms than physical capital because of its inalienable nature. The fall in the aggregate share of intangible assets coincided with a strong decline in aggregate employment and points towards a connection between the capital composition of firms’ balance sheet and aggregate labor markets. We investigate this relationship further in the next section by discussing the joint dynamics of asset intangibility, leverage and employment at the firm level.

\(^6\)The results are robust to various assumptions regarding the depreciation rate of intangible capital
Note: The figure shows the aggregate ratio of intangible to tangible assets from Compustat data. The sample includes all Compustat firm-year observations from 1980 to 2010 with positive values for sales, physical assets or total assets. Financial firms and utilities are excluded from the sample. The ratio of intangible-to-tangible assets is calculated using equations (1) and (2). Aggregate employment is from the BLS.

2.3 Asset Intangibility, Leverage and Employment at the Firm Level

In this section we test whether asset intangibility is a relevant factor in determining the amount of external financing at the firm level as well as explaining employment dynamics. In order to do this we construct a measure of leverage for each firm in the sample as the sum of debt in current liabilities (item DLC) and long-term debt (item DLTT) as a fraction of total assets (item AT).

We regress leverage as a function of the ratio of intangible to tangible assets for the whole sample and for the 2008-2009 crisis. We take one-year lagged values of asset intangibility to avoid endogeneity problems as investments and financing decisions are jointly determined. We also include firm-level and year fixed effects and other control variables such as market-to-book ratio, cash flow, acquisitions, and firm’s age. Table 1 shows the fixed-effect estimates of the panel regression for 19,900 firms and 187,566 firm-year observations. In line with the results by Falato et al. (2013), we find a negative relationship between asset intangibility and external debt financing, which implies that firms with larger shares of intangible assets have lower levels of leverage on average. The results also show that this effect was stronger during the last crisis, with a point estimate much larger than for the whole sample. Firms that had higher intangible-to-tangible ratios just prior to
the crisis experienced a higher reduction in external financing during 2008 and 2009.

Table 1: Asset Intangibility, Leverage and Employment

<table>
<thead>
<tr>
<th>Variables</th>
<th>Leverage</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Share Intangibles</td>
<td>$-0.0002$</td>
<td>$-0.0002$</td>
</tr>
<tr>
<td></td>
<td>[0.0001]</td>
<td>[0.0001]</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.096)</td>
</tr>
<tr>
<td>Share Intangibles $\times$ crisis 08</td>
<td>$-0.0359$</td>
<td>$-0.0030$</td>
</tr>
<tr>
<td></td>
<td>[0.0114]</td>
<td>[0.0015]</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>Market-Book</td>
<td>0.1976</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td>[0.0677]</td>
<td>[0.0003]</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Cash Flow</td>
<td>0.1486</td>
<td>-0.0005</td>
</tr>
<tr>
<td></td>
<td>[0.0921]</td>
<td>[0.0003]</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>Acquisition</td>
<td>$-1.0942$</td>
<td>0.0135</td>
</tr>
<tr>
<td></td>
<td>[0.5551]</td>
<td>[0.0150]</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.367)</td>
</tr>
<tr>
<td>Age</td>
<td>0.0233</td>
<td>0.2264</td>
</tr>
<tr>
<td></td>
<td>[0.0114]</td>
<td>[0.0456]</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.5622</td>
<td>6.0891</td>
</tr>
<tr>
<td></td>
<td>[0.1842]</td>
<td>[0.5086]</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Note: The table reports the fixed-effects panel estimates of asset intangibility (the share of intangible assets) and firm-level controls on leverage and employment for the 1980-2010 period using Compustat data. Robust standard errors clustered at the firm level are reported in brackets. P-values are presented in parentheses. Statistical significance is indicated by ***, ** and *, at the 1, 5 and 10 percent level, respectively. Leverage is the sum of debt in current liabilities (item DLC) and long-term (item DLTT) as fraction of total assets (item AT). Employment is the number of workers reported in item EMP in Compustat. The share of intangible capital is calculated using equations (1) and (2). The dummy crisis 08 takes the value of one for the years 2008-2009 and zero otherwise. The $R^2$ reported is the one between groups (firms). See the appendix for details on the construction of the control variables.

Finally, we test whether asset intangibility is related to employment dynamics. We regress
employment (item EMP) on the share of intangible assets, for the whole sample period and for the last crisis, and the same set of firm-level controls. We find a negative and significant relationship between asset intangibility and employment, in particular for the crisis period of 2008-2009. Firms with higher asset intangibility experienced larger declines in employment during the great recession.

These results suggest that it is important to understand the dynamics of firms’ asset composition in episodes where a credit crunch leads to a decline in economic activity and to a deterioration of labor market outcomes. In the next section we present a general equilibrium model in which asset tangibility plays a role in the business cycle investment and hiring decisions of financially constrained firms.

3 Model Description

3.1 Entrepreneurs

Entrepreneurs own capital $K_t$ and produce final goods combining the former with labor $L_t$. We distinguish between two types of capital, tangible and intangible, which we denote $K_{T,t}$ and $K_{I,t}$ respectively. This distinction will become relevant when discussing the entrepreneurs’ borrowing problem. The production technology exhibits constant returns to scale and is given by:

$$F(K_{T,t}, K_{I,t}, L_t) = Y_t = Z_t K_{T,t}^{\alpha_{K_T}} K_{I,t}^{\alpha_{K_I}} L_t^{1-\alpha_{K_T}-\alpha_{K_I}}$$

where $\alpha_{K_T}$ and $\alpha_{K_I}$ are the income shares of tangible and intangible capital respectively and $Z_t$ is an aggregate productivity shock.

Firms use some of their labor input to recruit new workers, thus dividing workers between two tasks: production ($L_t$) and recruiting ($v_t$). The law of motion for the total number of workers $n_t$ is given by:

$$n_{t+1} = (1 - x) n_t + \omega(\theta_t) v_t$$

where $x$ is an exogenous death shock and $\omega(\theta_t)$ is a function of the labor market tightness and represents the number of workers that a recruiter can hire. From the entrepreneurs’s point of view, next-period employment will be a function of the surviving matches this period and the amount of new workers that current recruiters are able to hire.

Entrepreneurs maximize their expected discounted flow of consumption $c^e_t$, by choosing how much to invest in physical capital, both tangible and intangible, how two divide workers between

---

7This specification follows Shimer (2010) and is consistent with the view that recruiting is a time-intensive activity. Our results are robust to the alternative specification in which recruiting costs are denominated in units of the final good (as in Mortensen and Pissarides).

8In Section 4.1 we discuss the calibration of the parameters governing the dynamics of the labor market. Given that in the data most of the employment fluctuations are driven by changes in the job finding rate, we don’t find the assumption of an exogenous separation rate restrictive.
production and recruiting activities and how much to borrow from households $B_{t+1}$. The optimization problem of the entrepreneur is thus summarized by:

$$\max_{C_t^{e}, K_{T,t+1}, K_{I,t+1}, v_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \gamma^t \log (c_t^e)$$

subject to the following budget constraint:

$$c_t^e + (1 + r_t)B_t + (K_{T,t+1} - (1 - \delta_T)K_{T,t}) + (K_{I,t+1} - (1 - \delta_I)K_{I,t}) + W_t n_t = Y_t + B_{t+1}$$

where $\delta_T$ and $\delta_I$ are depreciation rates of tangible and intangible capital respectively. The entrepreneurs are assumed to be less patient than households, such that their discount factor satisfies $\gamma < \beta$. Finally, entrepreneurs cannot commit to repaying their loans, and thus face the following borrowing constraint:

$$B_{t+1}(1 + r_{t+1}) \leq \chi_t K_{T,t+1} - W_t n_t$$

This constraint implies that entrepreneurs are able to borrow up to the point where the repayment equals a fraction $\chi_t$ of the total value of their tangible capital minus the wage bill. We assume here that intangible capital cannot be seized in the event of a default and thus cannot be included as part of the collateral. At the same time, the fact that we have the wage bill in the constraint ensures that even in the event of default in which the entrepreneur appropriates $1 - \chi_t$ of the collateral and liquidates the firm, there is enough pledgeable asset to pay the workers. The fraction of collateralized capital $\chi_t$, sometimes referred to as the “hair-cut”, evolves stochastically in this economy and reflects shocks to the terms of loans or current financial conditions. For example, a negative shock to $\chi_t$ implies that creditors are willing to lend less to the entrepreneur, relative to the same level of collateral. The timing of the constraint implies the following sequence of events: At the beginning of every period entrepreneurs repay their debt, $B_t (1 + r_t)$, and ask for a new loan, $B_{t+1}$ to finance production. Then they produce, invest in physical capital, pay workers, and consume. In equilibrium there is no default and entrepreneurs maximize their external financing, so the borrowing constraint always holds with equality.

Let $\mu_t$ be the multiplier associated to the borrowing constraint. The Euler equation for tangible capital is described by:

$$1 - \mu_t \chi_t = \gamma E_t \left[ \frac{c_t^e}{c_{t+1}^e} \left( r_{T,t+1} + (1 - \delta_T) \right) \right]$$

where $r_{T,t+1}$ is the return on tangible capital in terms of the final good defined by the standard capital-output ratio: $r_{T,t+1} = \alpha_{KT} \frac{Y_{t+1}}{K_{T,t+1}}$. Note that the multiplier associated to the borrowing constraint affects the optimal decisions regarding tangible capital directly. In particular, whenever

---

9. Intangible assets tend to be inalienable, having large value inside the firm but little value to creditors. This assumption is common in the literature on intangible capital (See Eissfeldt and Papanikolaou (2013), Chen (2014) and Falato, Kadyrzhanova, and Sim (2013)).
the borrowing constraint binds, it provides incentives for entrepreneurs to increase investment in tangible capital, so as to relax their financial constraint. The Euler equation of intangible capital is standard:

$$1 = \gamma E_t \left[ \frac{c^e_t}{c^e_{t+1}} (r_{I,t+1} + (1 - \delta_I)) \right]$$

where $r_{I,t+1}$ is the return on intangible capital, defined as: $r_{I,t+1} = \alpha K_t \frac{Y_{t+1}}{K_{t+1}}$.

The Euler equation for debt can be expressed as:

$$\mu_t = \frac{1}{1 + r_t + 1} - \gamma E_t \frac{c^e_t}{c^e_{t+1}}$$

Given the differences in discount factors between the representative household and the entrepreneur, the multiplier $\mu_t$ is positive in steady-state, which implies that in steady-state the borrowing constraint is satisfied with equality.

Regarding the decision of assigning workers between productive labor and recruiting, the following Euler equation describes the trade-off faced by the entrepreneur:

$$\frac{MPL_t}{\omega(\theta_t)} = \gamma E_t \left[ \frac{c^e_t}{c^e_{t+1}} \left( MPL_{t+1} \left( 1 + \frac{1 - x}{\omega(\theta_{t+1})} \right) - W_{t+1} (1 + \mu_{t+1}) \right) \right]$$

where $MPL_t$ is the marginal product of labor in terms of the final good.

The left-hand side of the equation represents the cost of hiring one extra worker, which implies increasing the number of recruiters today, at the expense of productive labor. As we indicated before, the function $\omega(\theta_t)$ indicates the number of workers that a recruiter can hire so its inverse, $1/\omega(\theta_t)$, represents the number of recruiters needed to hire one worker. Thus, the left-hand side of the Euler equation is equal to the value of foregone production associated with increasing the number of recruiters, so as to increase the number of production workers by one. The right-hand side of the equation is the expected discounted return of increasing the size of the firm by one worker, which is the marginal product of labor net of wage costs. In particular, the marginal product of labor includes not only the new worker hired in period $t$ but also the workers freed from recruiting in $t + 1$ while allowing the firm to maintain its original size in $t + 2$. With respect to the wage costs, note that the increase in the firm’s payroll includes the wage of the newly hired worker plus the shadow cost in terms of external funds.

### 3.2 Households

There is a representative household composed by a unit measure of infinitely lived individuals with the same preferences over consumption and disutility of working. In every period, each member of the household can be either employed or unemployed. The disutility of working is represented by $\varphi$ and in equilibrium there is a fraction $n_t$ of family members that are employed.
The representative household supplies funds to entrepreneurs in the form of non-contingent one-period bonds, $D_t$. There is a household-level budget constraint, which states that the consumption of all family members, $c_t$, and the amount of funds supplied to the firm should be equal to the total labor income of the household plus the return on the previous period bond holdings.

The optimization problem of the representative household can be written as:

$$\max_{c_t, D_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) - \varphi n_t \right]$$

subject to the budget constraint

$$c_t + D_{t+1} = n_t W_t + (1 + r_t) D_t$$

and the law of motion of the fraction of employed workers given by:

$$n_{t+1} = (1 - x) n_t + f_t(\theta_t) (1 - n_t)$$

where $W_t$ is the wage rate, $r_t$ is the interest rate on loans determined in the bond market, and $f_t$ is the job finding rate, which depends on the labor market tightness, $\theta_t$. The law of motion indicates that employment in the next period will be determined by this period’s surviving matches plus the new matches formed in the labor market. The first-order optimal conditions for the household problem are summarized in the following Euler equation:

$$\frac{1}{c_t} = \beta E_t \left[ \frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

### 3.3 Wage Determination

At the start of every period, each employed worker bargains with his employer over his wage. If bargaining fails, the match breaks up, the worker is unemployed in the current period and the firm loses the job. If bargaining succeeds, the worker is paid the bargained wage this period and is put to work by the firm either producing output or recruiting other workers. Moreover, the worker and the firm anticipate agreeing on the equilibrium wage in any future period when they are matched. In equilibrium, the bargained wage equals the equilibrium wage in every period.

Following most of the literature, we use the Nash Bargaining solution to characterize the wage.\(^{10}\) Solving for the equilibrium wage gives us the following expression (see the Appendix for the full derivation):

\(^{10}\)As in Shimer (2010), we assume that firms don’t internalize that investment and hiring decisions affect wages through changes in the marginal product of labor. We study the quantitative implications of departing from this assumption in an online appendix. In order to solve for the equilibrium wages assuming intra-firm bargaining, we follow Cahuc and Wasmer (2001). We find that after re-calibrating the steady-state of the model, this specification delivers business cycle statistics for aggregate variables almost identical to our benchmark. This result is in line with the findings by Krause and Lubik (2007).
\[ W = \phi \frac{MPL}{(1+\mu)} \left( 1 + \theta \frac{\beta}{\gamma} + \frac{1 - \phi}{\omega(\theta)} \left( 1 - \frac{\beta}{\gamma} \right) \right) + (1 - \phi) \varphi c \]

where \( \phi \in (0, 1) \) is the workers’ bargaining power.

As in the standard search model, equilibrium wages are determined as a weighted average of two terms, the marginal product of labor (of the additional worker and of the workers released from recruiting by his employment) and the marginal rate of substitution between leisure and consumption. In the standard model, if a worker is employed, he can directly produce \( MPL \) units of output and, by marginally reducing the unemployment rate, frees up \( \theta \) workers to produce output rather than recruit, since \( \theta \) is the aggregate recruiter-unemployment ratio. That is, these additional workers can produce while keeping the recruiter-unemployment ratio constant. Thus, the first term of the equilibrium wage is given by \( \phi MPL (1 + \theta) \). In our setup, the wedge between the discount factors of household and firms and the shadow cost of financing labor that results from the working capital assumption give rise to some differences relative to the standard specification. In particular, in our equilibrium wage the marginal product of labor is adjusted by the financing cost of labor \( 1 + \mu \). Additionally, the fact that firms are relatively more impatient than households introduces two further differences. As mentioned earlier, at the aggregate level, the employment of an additional worker releases \( \theta \) workers from recruiting, while keeping the recruiter-unemployment ratio constant. At the same time, at the firm level, increasing the size of the firm by one worker, frees up resources from recruiting by an amount equivalent to \( \frac{1 - \varphi}{\omega(\theta)} \) workers. As firms and households discount the future at different rates, the present value associated to both of these savings is valued differently by the two types of agents. In the case of the recruiter-unemployment ratio, this determines that \( \theta \) appears multiplied by the the ratio of discount factors \( \frac{\beta}{\gamma} \). Regarding the saving at the firm level, the difference in the level of impatience determines that we have a new term in our wage expression, relative to the standard specification, given by \( \frac{1 - \varphi}{\omega(\theta)} \left( 1 - \frac{\beta}{\gamma} \right) \). Note that if there was no wedge between \( \beta \) and \( \gamma \), this term would disappear.

### 3.4 Equilibrium

An equilibrium in this economy consists of sequences of prices \( \{r_t, W_t\}_{t=0}^{\infty} \) and allocations for the households and entrepreneurs such that, given prices, initial conditions, the borrowing constraint of the entrepreneur and the stochastic processes, these allocations are optimal, as defined by the first-order conditions mentioned before, and labor and debt markets clear:

\[ f_t(\theta_t) (1 - n_t) = \omega(\theta_t) v_t \]

\[ D_t = B_t \]

Finally, we assume a constant returns to scale matching function in which the elasticity with
respect to the market tightness is the same as the bargaining power of workers:

\[ m_t = \bar{\omega} u_t v_t^{1-\phi} \]

where \( m \) is the number of matches and \( u \) is the number of unemployed workers in the labor market. Under this assumption the recruiting cost function can be expressed as:

\[ \omega(\theta) = \bar{\omega} \theta^{-\phi} \]

and the labor market tightness is defined by:

\[ \theta = \frac{v}{1-n} \]

### 3.5 Steady-State

It is easy to verify that aggregate productivity plays no role in steady state because of the assumptions on preferences consistent with a balanced-growth path. On the contrary, the fraction \( \chi \) has a first-order effect on market tightness and all other endogenous variables of the model. In section (7.3) of the appendix we derive the market-clearing condition for labor market in the steady state. From this expression it can be shown that \( \theta \) depends negatively on \( \chi \). Intuitively, a higher \( \chi \) implies that higher debt can be sustained in equilibrium. As entrepreneurs become less constrained, they can borrow and produce more, thus using more labor which reduces its marginal product in steady state. On the household side, the higher equilibrium debt implies that a higher fraction of household income is unrelated to the employment status of its members. Overall, when we consider a higher fraction of pledgeable collateral in steady state the total surplus of a match is reduced.\(^{11}\)

### 4 Quantitative Analysis

In this section we calibrate and simulate the model to study the response of the real economy when hit by productivity and financial shocks estimated from the data\(^{12}\).

#### 4.1 Calibration

Following most of the literature on search models, we calibrate the model on a monthly basis. Table 2 presents the complete list of parameter values used in our numerical simulation. The discount factor of the representative household, \( \beta \), is set to 0.996, implying a real annual interest rate of

---

\(^{11}\)An alternative way to see this effect is by looking at the steady-state replacement ratio of workers defined as \( \frac{C(1+\mu)}{MPL} \). The higher the value of the multiplier \( \mu \), which is given in steady-state by the difference between \( \beta \) and \( \gamma \), the higher the replacement ratio in the model.

\(^{12}\)We solve the model with standard perturbation methods using a first-order approximation around the non-stochastic steady state.
### Table 2: Benchmark Calibration: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source / Target</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household’s discount factor</td>
<td>5% Annual Interest Rate</td>
<td>( \beta )</td>
<td>0.996</td>
</tr>
<tr>
<td>Workers’ disutility of work</td>
<td>Unempl. Rate 5%</td>
<td>( \varphi )</td>
<td>0.917</td>
</tr>
<tr>
<td>Entrepreneur’s discount factor</td>
<td>Quarterly Debt-to-GDP of 3.36</td>
<td>( \gamma )</td>
<td>0.94</td>
</tr>
<tr>
<td>Unconditional mean of financial shock</td>
<td>Quarterly Debt-to-GDP of 3.36</td>
<td>( \bar{\chi} )</td>
<td>0.85</td>
</tr>
<tr>
<td>Depreciation of tangible capital</td>
<td>Annual depreciation of 8%</td>
<td>( \delta_T )</td>
<td>0.007</td>
</tr>
<tr>
<td>Depreciation of intangible capital</td>
<td>Annual depreciation of 20%</td>
<td>( \delta_I )</td>
<td>0.017</td>
</tr>
<tr>
<td>Income share of tangible capital</td>
<td>Corrado et al. (2009)</td>
<td>( \alpha_{K_T} )</td>
<td>0.25</td>
</tr>
<tr>
<td>Income share of intangible capital</td>
<td>Corrado et al. (2009)</td>
<td>( \alpha_{K_I} )</td>
<td>0.15</td>
</tr>
<tr>
<td>Workers’ bargaining power and Elasticity</td>
<td>Shimer (2010)</td>
<td>( \phi )</td>
<td>0.5</td>
</tr>
<tr>
<td>Exogenous separation rate</td>
<td>Shimer (2010)</td>
<td>( x )</td>
<td>0.03</td>
</tr>
<tr>
<td>Matching function efficiency</td>
<td>Shimer (2010)</td>
<td>( \bar{\omega} )</td>
<td>2.32</td>
</tr>
</tbody>
</table>

5%. The disutility parameter \( \varphi \) is set to match the long-run average unemployment rate of 5%.$^{13}$ The income shares of tangible and intangible capital, \( \alpha_{K_T} \) and \( \alpha_{K_I} \), are set equal to 0.25 and 0.15, following estimates by Corrado et al. (2009). The depreciation of tangible capital, \( \delta_T \), is standard (0.08 in annual terms), while for the depreciation rate of intangible capital, we follow Eisfeldt and Papanikolaou (2013) and set it equal to 0.20 in annual terms.

The entrepreneur’s discount factor, \( \gamma \), is set equal to 0.94, so as to match, together with the mean value of the share of collateralized capital, \( \bar{\chi} \), a steady state ratio of debt to quarterly GDP of 3.36 (in quarterly terms). This is the average ratio over the period 1980.I-2010.II for the non-financial business sector based on data from the Flow of Funds (for debt) and National Income and Product Accounts (for GDP). The mean value of the share of collateralized capital is computed by assuming that the borrowing constraint is always binding. We first compute \( \chi_t \) as a residual using empirical series for end of period debt (Flow of Funds), tangible capital (NIPA and own calculations) and wage bill (also from NIPA), all relative to GDP and transformed into monthly frequency. Then, \( \bar{\chi} \) is determined as the average of this residual over the sample period.

Regarding the workers’ bargaining power, the exogenous separation rate and the parameter governing the matching efficiency we follow the literature, in particular the parameterization of Shimer (2010), and set \( \phi \) equal to 0.5, \( x \) equal to 0.03 and \( \bar{\omega} \) equal to 2.32.

$^{13}$The replacement ratio of this calibration is 0.5, which is relatively low compare with the literature and guarantees that the volatility of employment is not achieved by having a small steady-state surplus of the match between workers and firms.
4.2 Stochastic Processes

The economy is subject to both aggregate productivity shocks $z_t$ and financial shocks, that is, shocks to the share of collateralized capital $\chi_t$. We use the standard solow residual approach to recover $z_t$ from the data. Note that we use quarterly data from 1952.I to 2010.II to construct the series of shocks, and once we have the quarterly innovations, we transform them into monthly to analyze the dynamics of the model induced by these shocks. Data for output and investment is obtained from the NIPA accounts. In particular, we use the category of intellectual property investment to define intangible investment in our model, so that tangible investment is then defined as all non-residential investment minus intellectual property. We use the perpetual inventory method to construct the series for tangible and intangible capital (initial capital is computed as investment in 1952.I divided by the depreciation rate, however, given that our period of study goes from 1980.I to 2010.II the initial values do not affect our results). Finally, labor data is obtained from the BLS. From the production function we get:

$$\hat{z}_t = \hat{y}_t - \alpha_K \hat{k}_{T,t} - \alpha_K \hat{k}_{I,t} - (1 - \alpha_K - \alpha_K)\hat{l}_t \quad (3)$$

where hat variables are percentage deviations from trend. Given the values for the parameters $\alpha_K$ and $\alpha_K$ and empirical series for $\hat{y}_t$, $\hat{k}_{T,t}$, $\hat{k}_{I,t}$ and $\hat{l}_t$ we can construct the $\hat{z}_t$ series.

To construct the series for the financial shock, we assume that the borrowing constraint holds and write all the variables relative to output:

$$\frac{B_{t+1}(1 + r_{t+1})}{Y_t} + \frac{W_{t}n_{t}}{Y_t} = \chi_t \frac{K_{T,t+1}}{Y_t} \quad (4)$$

Each term in this equation has a clear counterpart in the data. The first term is the leverage of the private sector, which can be computed using the Flow of Funds data. The second term is the labor-share of the economy which can be measured using the NIPA accounts. The right-hand side term is the tangible capital -output ratio which can also be computed using data from the NIPA accounts. We can thus compute the series for $\chi_t$ as a residual\(^{14}\). Figure 2 plots the series for both shocks over the period 1980.I-2010.II.

\(^{14}\)As in Jermann and Quadrini (2012), the validity of this procedure depends on the validity of the assumption that the borrowing constraint is always binding. We check this condition ex-post by feeding the constructed series into the model and checking whether the constraint is always binding.
Figure 2: TFP and Financial Series

Note: The graph plots the TFP ($\hat{z}_t$) and Financial Shocks ($\chi_t$) computed using equations (3) and (4), respectively. It also plots in the right axis (inverted scale) the net percentage tightening of lending standards from the Senior Loan Officer Survey on Bank Lending conducted by the Federal Reserve Board: http://www.federalreserve.gov/boarddocs/snloansurvey/

Finally, we estimate the autoregressive system:

$$
\begin{pmatrix}
\hat{z}_{t+1} \\
\hat{\chi}_{t+1}
\end{pmatrix} =
\begin{pmatrix}
\rho_{zz} & \rho_{z\chi} \\
\rho_{\chi z} & \rho_{\chi\chi}
\end{pmatrix}
\begin{pmatrix}
\hat{z}_t \\
\hat{\chi}_t
\end{pmatrix} +
\begin{pmatrix}
\epsilon_{z,t+1} \\
\epsilon_{\chi,t+1}
\end{pmatrix}
$$

(5)

where $\epsilon_{z,t+1}$ and $\epsilon_{\chi,t+1}$ are iid shocks with standard deviations $\sigma_z$ and $\sigma_\chi$ respectively. The results for the estimated parameters governing the stochastic properties of the shocks are presented in Table 3.
4.3 Dynamics Induced by Shocks

To study the dynamics of the model induced by the constructed series of shocks, we conduct the following simulation. Starting with initial values of $\hat{z}_{1980,I}$ and $\hat{\chi}_{1980,I}$ we compute the quarterly innovations (recall that we use quarterly data to construct the series of shocks) for the period going up to 2010.II. These are shown in Figure 3. It is important to point out how the decline in $\hat{\chi}$ during the last crisis is the largest in all of our sample period, and it is in this sense that the recent crisis is characterized by the most severe financial conditions experienced in the US economy for decades.

Once we have the quarterly innovations, we transform them into monthly, which we feed into the model and compute the responses for output, employment, unemployment rate, debt and all other variables in the model.\textsuperscript{15} We do this for three different specifications of the model, one in which

\textsuperscript{15}Appendix 7.4 presents the details in how monthly estimates are derived from the quarterly data.

---

Table 3: Stochastic Properties of Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation productivity shock</td>
<td>$\sigma_z$</td>
<td>0.0036</td>
</tr>
<tr>
<td>Standard deviation financial shock</td>
<td>$\sigma_\chi$</td>
<td>0.0042</td>
</tr>
<tr>
<td>Covariance parameter innovations</td>
<td>$\sigma_{z\chi}$</td>
<td>0.0004</td>
</tr>
<tr>
<td>Autoregressive parameter for productivity shock</td>
<td>$\rho_{zz}$</td>
<td>0.9574</td>
</tr>
<tr>
<td>Autoregressive parameter for financial shock</td>
<td>$\rho_{\chi\chi}$</td>
<td>0.9827</td>
</tr>
<tr>
<td>Spill-over from financial shock to productivity</td>
<td>$\rho_{\chi z}$</td>
<td>-0.0199</td>
</tr>
<tr>
<td>Spill-over from productivity to financial shocks</td>
<td>$\rho_{z\chi}$</td>
<td>0.1186</td>
</tr>
</tbody>
</table>

Note: The table reports the stochastic properties of the TFP ($\hat{z}_t$) and Financial Shocks ($\hat{\chi}_t$) from equation (5)
we only have productivity shocks (\( \hat{\chi} \) is kept constant at its unconditional mean value \( \bar{\chi} \)), one with financial shocks only and one with both. Note that although we use the actual sequence of shocks, the agents do not perfectly anticipate them, but forecast their future values using the autoregressive system described earlier. Finally, we verify that the multiplier of the borrowing constraint remains positive during the whole simulation period, implying that the constraint is always binding.

Figure 4 plot the response of output, employment, the unemployment rate and the debt to GDP ratio under the three different specifications together with the corresponding data. \(^{16}\) Regarding output, TFP shocks capture most of its dynamics. Financial shocks do not generate enough output fluctuations during the first years of the sample, as a result of smaller innovations during the 1980s (see figure 3), but appear to perform better during the second half. However, during the great recession financial shocks predict a smaller decline in production relative to what was observed in the data. The specification with both shocks tracks output fluctuations quite well, though slightly overestimating its volatility at the end of the sample period.

With respect to employment, TFP shocks alone do not capture any of the movements observed in the data, as they fail to generate enough volatility. Moreover, the drop in employment captured by TFP shocks during the last recession is barely noticeable relative to the fall observed in the data. On the contrary, the specification with financial shocks is capable of replicating the behavior of employment during almost all of the sample period. In particular, financial shocks generate sharp drops in employment in three of the four recessions: 1990-1991, 2001 and 2008-2009. Similar results hold for the unemployment rate. Financial shocks generate enough movement in the job finding rate such that the model can replicate the cyclical volatility of unemployment and, in particular, the large increase in the unemployment rate during the great recession. The model with financial shocks also fits the dynamics of the debt-to-GDP ratio of the economy. This is not the case for the model with productivity shocks, which fails to generate the necessary volatility to explain the behavior of either the unemployment rate and the amount of leverage in the economy.

\(^{16}\)All variables are in log deviations from trend over the period 1980.I-2010.II, with the exception of the unemployment rate which is plotted as absolute deviations.
4.4 Impulse-Responses and RBC Statistics

In order to understand the mechanisms behind the good fit of the model we analyze its behavior when hit by a one standard deviation negative shock to aggregate productivity and to the parameter governing the fraction of collateralized asset. Figure 5 shows the impulse response functions.

We first look at the effects of a TFP shock. This shock impacts output directly, and is thus capable of generating a larger and more persistent fall in output relative to a financial shock. However, the response of other variables such as employment, aggregate capital stock and debt over a long horizon to a TFP shock is relatively weak. The small decline in employment is not surprising in light of two results. First, Shimer (2005, 2010) neutrality result, that predicts low volatility of employment when changes in productivity affect in a similar way both consumption and the marginal product of labor, leaving the surplus of a match unchanged. Second, as shown in
Olivella and Roldan (2011), in models with reproducible capital general equilibrium effects dampen the response of real variables to productivity shocks, even in the presence of financial frictions.

Regarding relative variables, a TFP shock has a negligible effect on the composition of capital. Changes in TFP affect all production inputs uniformly and have little reallocation effects between the two types of capital.

Figure 5: Impulse Responses: TFP and Financial Shock

Note: The graph plots the impulse response function of the main variables of the benchmark to a one-standard deviation productivity ($z_t$) and financial shock ($\chi_t$).

Financial shocks have a very different impact on all variables. They produce a larger drop in employment and in the stock of debt, while generating a much higher increase in the multiplier of the borrowing constraint, where they have a first order effect.

The high volatility of employment in the model generated by financial shocks can be explained as follows. On the production side, a tighter borrowing constraint forces entrepreneurs to reduce their debt and scale down their operation. It also gives them incentives to reallocate resources towards the pledgable asset $K_T$ at the expense of investment in the non-collateralized capital $K_I$, so as to offset the tighter borrowing constraint. The decrease in $K_I$ is such that aggregate capital (the sum of $K_I$ and $K_T$) also falls, reducing the marginal product of labor and thus the incentives to hire workers. On the cost side, as a result of the need for working capital, the tightening of the

\footnote{This is consistent with the decline of the intangible-to-tangible asset ratio during the great recession that we document in section 2.}
borrowing constraint generates an increase in the shadow cost of financing labor, providing further incentives for firms to cut back employment. This appears in the model as a labor wedge. Overall, the cost of labor falls by less than its marginal product, reducing the value of a match surplus and causing a significant decrease in employment.\footnote{Note that our financial shock is different from an investment price shock, mainly in that the latter results in a significant increase in investment and capital, causing the marginal product of labor to increase. This, together with the fact that the shadow cost of financing labor remains relatively unchanged, results in employment moving significantly less than when a financial shock hits.}

Having two types of capital is key for obtaining these results, and we explain this in more detail in our sensitivity analysis section. Table 4 presents the standard Real Business Cycle statistics for selected variables for the data, for our benchmark model with both financial and TFP shocks and for model specifications that only include one shock at the time. The volatility of the data is computed as the standard deviation of the cyclical component of the detrended quarterly series.\footnote{The sample period is the same used for estimating the stochastic processes: 1980.I-2010.II. We use data from NIPA accounts and report statistics using the Hodrick-Prescott (HP) filter with the standard smoothing parameter $\lambda = 1,600$ for quarterly data. The volatility for $\theta$ is taken from Shimer (2010).}

To compute volatilities implied by the model we follow the same procedure but adjust the HP filter to control for the fact that the model is calibrated at a monthly frequency.\footnote{The statistics for the model are the theoretical moments using an smoothing parameter of $129600$ as suggested by Ravn and Uhlig (2002).}

The statistics for the model with both shocks and the two alternative specifications with only one shock confirm what we discussed earlier when analyzing the IRF of the model. In particular, the model with a TFP shock fails to reproduce the volatility of employment variables. Moreover, these statistics confirm that the introduction of borrowing constraints has no effect on the typical successes and failures of the standard search model.

The financial shock however, produces statistics that are significantly different. Volatility of employment is almost four times higher than in the model with TFP shocks, so that financial shocks explain almost 90% of the volatility of the data. Aggregate capital stock relative to output

\begin{table}[h]
\centering
\caption{Theoretical Second Moments of Model}
\begin{tabular}{lcccc}
\hline
 & \multicolumn{3}{c}{Model} \\
 & Both Shocks & TFP Shocks & Financial Shocks \\
\hline
Volatility Relative to Output \\
Employment & 0.66 & 0.42 & 0.15 & 0.59 \\
Capital-Output & 1.38 & 1.29 & 0.96 & 1.56 \\
Market Tightness & 15.30 & 17.41 & 6.45 & 26.15 \\
\hline
Correlations with Output \\
Employment & 0.79 & 0.89 & 0.96 & 0.80 \\
Investment & 0.38 & 0.98 & 0.95 & 0.93 \\
\hline
\end{tabular}
\end{table}
and the market tightness are also more volatile than their counterpart with TFP shocks, though
the latter is a bit too volatile relative to the data. This model also delivers the correct correlation
between employment and output, while overestimating that of investment and output.

The model that includes both shocks (with properties as estimated in section 4.2) is able to
deliver statistics closer to the data for the case of capital and market tightness, but slightly lower
for employment. The market tightness volatility, relative to output, is 17.41 in the model with both
shocks, close to the 15.3 of the data and almost 3 times bigger than the volatility delivered by the
standard search model with TFP shocks.

5 Sensitivity Analysis

In this section we analyze the importance of some of our main assumptions for our results. In
particular, we look at the role of two capitals, by comparing our model with different versions
of a model with only one capital, and we eliminate the wage bill from the borrowing constraint,
in each case studying the impact of financial shocks on the volatility of output, employment and
unemployment. We also recalibrate the model to have lower or higher leverage in equilibrium and
check how this changes our results.

5.1 The role of intangible capital

We begin by analyzing the role of having two types of capital. To do so, we consider 3 alternative
versions of our model, all of them with only one type capital and compare them to our benchmark.
The first of these alternative models is exactly as our benchmark, with the only difference being
that there is only one type of capital in the economy, and all of it can be used as collateral. The
second model is that of Jermann and Quadrini (2012) which differs from our one capital version
mainly in that they don’t consider search frictions and in the type of borrowing constraint. Finally,
for the third model we take Jermann and Quadrini (2012) and add labor market frictions. We will
refer to these models as model (i), (ii) and (iii) respectively. We hit all of these models and our
benchmark with a one standard deviation negative financial shock and show the results in Figure
6.
The impulse response functions of the different models analyzed show that having intangible capital is key to get significant unemployment volatility and a big drop in output with financial shocks. Consider first model 1K (i), in which there is only one type of capital and all of it can be used as collateral. Under the latter model specification a financial shock has a hard time generating a recession and a significant decrease in employment. As in our benchmark specification, when a financial shock hits the economy, the entrepreneur has incentives to increase the pledgeable asset, so as to counteract the tightening of the borrowing constraint. However, given that in this setup there is only one type of capital, the reallocation effect between the pledgeable and non pledgeable asset that is present in the benchmark model is absent. Thus, a financial shock results in a smaller reduction in the capital stock after the shock, which in turn leads to a smaller decrease in the marginal product of labor, employment and output.

Note that after the shock, debt falls by more in model (i) than in the benchmark model. This is explained by the fact that in this economy pledgeable capital also falls by more, as entrepreneurs cannot finance the accumulation of the pledgeable asset by freeing up resources when cutting down investment in intangible assets, as in the benchmark case.

Having stated the importance of having intangible capital in our environment, we compare our benchmark model with the environment studied in Jermann and Quadrini (2012), that we label
No Search (ii). In model (ii) there is only one capital (physical capital), there are no frictions in labor markets, firms face an additional intra-temporal financing constraint and there are adjustment costs for changes in equity payouts. Figure 6 shows that output falls less in model (ii) than in our benchmark while the behavior of employment is very similar. Why does this specification with one capital appear to work well at explaining employment volatility? The explanation lies on the fact that in model (ii) the only variables that can adjust in the intra-period when the economy is hit by a financial shock are labor and dividends because capital and debt are given. In other words, if firms want to keep the production plan unchanged, a negative financial shock requires a reduction in equity payout. Because the number of workers can be adjusted freely, while dividends are subject to quadratic costs of adjustment, firms prefer to cut down on employment, and so labor absorbs most of the shock. In the subsequent periods, when debt and capital can adjust, there is a strong deleveraging of the economy -debt in this model is more volatility than in the benchmark and the data- while the decline in the capital stock is only mild because firms have incentives to invest in physical capital because of its role as collateral.

There is one caveat to this specification, though. This model, as any other model with endogenous labor supply and no frictions in the labor market, generates a large decline in employment by producing a large drop in wages, inducing workers to supply less labor. Therefore, it cannot explain unemployment volatility, which is a key element in the discussion of labor markets during the last financial crisis.

This brings us to model (iii), 1K-Alternative Friction, in which we add labor market frictions to model (ii). In this case we see that output and employment increase instead of falling, and unemployment falls. Thus a financial shock is not able to generate a recession in this setting but on the contrary, generates an expansion. Thus, the same argument we made for our model (i) can be made for this model. In a model with only one type of capital, and in which the firm cannot change its demand for labor immediately (given search frictions), a financial shock generates incentives to increase capital to offset the tighter financial constraint, leading to a smaller decline in capital, output and employment than otherwise and in some cases, like model (iii), to an outright expansion. Model (iii) thus shows that the model in Jermann and Quadrini (2012) with one capital can reproduce the volatility of real variables only if there are no search frictions.

5.2 Wage bill

If labor costs are not part of the borrowing constraint, hiring decisions are not directly affected by the financial situation of the firm, though they will still be affected indirectly, through capital and borrowing decisions. In this case the model exhibits around 85% of the volatility of employment observed in the benchmark model (the volatility of employment is 0.51 vs to 0.59 in the benchmark). That is, roughly one sixth of the volatility generated by the financial shock in the benchmark model
is because of the direct effect of the tightening of the borrowing constraint on the shadow cost of labor in terms of financing.

5.3 Leverage

Finally, we consider a calibration with a higher leverage in equilibrium. As discussed in subsection 3.5, the elasticity of employment resulting from financial shocks is directly related to the level of leverage in the economy. If we re-calibrate $\chi$ and $\gamma$ to be consistent with a debt-to-gdp ratio twice as big as the benchmark case the volatility of employment doubles too.

6 Conclusions

In this paper we study the effect of exogenous shocks to financing conditions on capital allocation and labor market outcomes. We find that, unlike aggregate productivity shocks, in an economy with two types of capital and in which only one can be used as collateral, financial shocks can replicate the large fluctuations in aggregate employment and other labor market variables observed in the data. This suggests that understanding cyclical changes in credit and the composition of pledgable and non-pledgable assets is important for business cycle dynamics. Given our representative firm assumption, understanding how financial shocks reallocate resources among heterogeneous producers depending on their capital composition is beyond the scope of this paper but seems a promising area for future research.\textsuperscript{21}

\textsuperscript{21}Recent papers by Khan and Thomas (2013) and Buera et al. (2015) discuss financial shocks in the context of heterogenous producers but without any asset heterogeneity.
References


27


7 Appendix

7.1 Firm-level Data and Control Variables

Section 2 discusses the relationship between asset intangibility, measured as the ratio of intangible-to-tangible assets, leverage and employment using firm-level Compustat data. The firm-level control variables are defined as follows: Market-to-Book is the ratio of total assets (item AT) minus the value of equity (item CEQ) as fraction of total assets. Cash Flow is income before extraordinary items (item IB) minus dividends (item DVT) divided by total assets. Acquisitions is the ratio of acquisitions (item AQA) to total assets. Age is based on the first appearance of the firm in the sample or in the IPO date reported by the company.

7.2 Wage Determination

We present here the characterization of the wage using the Nash bargaining solution. This solution implies that the bargained wage maximizes the following Nash product

\[ W = \argmax \left( \tilde{V}_n \right) ^\phi \left( \tilde{J}_n \right) ^{1-\phi} \]

where \( \tilde{V}_n \) is the marginal value for a household of having a worker employed at wage \( W \) and \( \tilde{J}_n \) is the marginal value of this worker for the firm. The bargaining power of workers is represented by \( \phi \).

Taking logs to the Nash product and deriving with respect to the equilibrium wage \( \bar{W} \), we get the following expression

\[ \phi \frac{\tilde{V}_n'(W)}{\tilde{V}_n(W)} + (1 - \phi) \frac{\tilde{J}_n'(W)}{\tilde{J}_n(W)} = 0 \]

Using a recursive representation for the household’s problem, we compute the marginal value of an employed member at the equilibrium wage

\[ V_n(D, n) = \frac{\bar{W}}{c} - \varphi + \beta(1 - x - f(\bar{\theta})) EV_n(D', n') \]

We also compute the marginal value of having a worker employed at an arbitrary wage \( W \) for one period and the equilibrium wage thereafter

\[ \tilde{V}_n(W) = \frac{W - \bar{W}}{c} + V_n(D, \bar{n}) \]

where the first term is the incremental income from receiving the arbitrary wage rather than the equilibrium wage \( W - \bar{W} \), measured in terms of marginal utility and the second term is the marginal value of having a worker employed at the equilibrium wage.

We next turn to the firms. Expressing their problem recursively, and assuming an interior solution for the share of recruiters \( \nu = \frac{x}{n} \), we obtain the following first order condition for \( \nu \)
\[ MPL = \omega(\theta) \gamma EJ_n(n') \]

The envelope for employment is

\[ J_n(n) = MPL(1 - \nu) - W(1 + \mu) + (1 - x + \nu \omega(\theta))\gamma EJ_n(n') \]

We can rewrite the first order condition

\[ EJ_n(n') = \frac{MPL}{\omega(\theta)\gamma} \]

and plug it in the envelope to eliminate the continuation value

\[ J_n(n) = MPL(1 - \nu) - W(1 + \mu) + (1 - x + \nu \omega(\theta))\gamma \frac{MPL}{\omega(\theta)\gamma} \]

Simplifying, we obtain

\[ J_n(n) = MPL(1 + \frac{1 - x}{\omega(\theta)}) - W(1 + \mu) \]

Finally, the marginal profit for a firm of employing a worker at an arbitrary wage \( W \) and at the equilibrium wage \( \bar{W} \) thereafter is given by

\[ \bar{J}_n(W) = (\bar{W} - W) (1 + \mu) + J_n(\bar{\theta}) \]

We now plug in expressions obtained for: \( \bar{V}_n(\bar{W}), \bar{V}'_n(\bar{W}), \bar{J}_n(\bar{W}) \) and \( \bar{J}'_n(\bar{W}) \)

\[ \phi \bar{V}'_n(\bar{W}) + (1 - \phi) \frac{J_n(\bar{W})}{J_n(\bar{\theta})} = \phi \frac{\bar{V}_n(D,\bar{\theta})}{\bar{V}_n(D,\bar{\theta})} + (1 - \phi) \frac{(1 + \mu)}{J_n(\bar{\theta})} = 0 \]

\[ \phi J_n(\bar{\theta}) = (1 - \phi)(1 + \mu)\bar{c}V_n(D,\bar{\theta}) \]

Replacing \( V_n(D,\bar{\theta}) \) we get

\[ \phi J_n(\bar{\theta}) = (1 - \phi)(1 + \mu)\bar{c} \left[ \frac{\bar{W}}{\bar{c}} - \varphi + \beta(1 - x - f(\bar{\theta}))E\bar{V}_n(D',n') \right] \]

and further replacing \( \bar{V}_n(D',n') \)

\[ \phi J_n(\bar{\theta}) = (1 - \phi)(1 + \mu)\bar{c} \left[ \frac{\bar{W}}{\bar{c}} - \varphi + \beta(1 - x - f(\bar{\theta}))E\frac{\phi J_n(n')}{(1 - \phi)(1 + \mu)\bar{c}} \right] \]

Simplifying

\[ \phi J_n(\bar{\theta}) = (1 - \phi)(1 + \mu)\bar{W} - (1 - \phi)(1 + \mu)\bar{c}\varphi + \beta(1 - x - f(\bar{\theta}))E\phi J_n(n') \]

Using the first order condition for \( \nu \) and envelope for employment we replace \( J_n(\bar{\theta}) \) and \( EJ_n(n') \)

\[ \phi \left( MPL(1 + \frac{1 - x}{\omega(\theta)}) - \bar{W}(1 + \mu) \right) = (1 - \phi)(1 + \mu)\bar{W} - (1 - \phi)(1 + \mu)\bar{c}\varphi + \beta(1 - x - f(\bar{\theta}))\frac{MPL}{\omega(\theta)\gamma} \]

Simplifying and solving for the equilibrium wage, we get

\[ \bar{W} = \phi \frac{MPL}{(1 + \mu)} \left( 1 + \bar{\theta} \frac{\bar{c}}{\gamma} + \frac{1 - x}{\omega(\theta)} \left( 1 - \frac{\bar{c}}{\gamma} \right) \right) + (1 - \phi)\varphi \bar{c} \]
7.3 Labor Market Clearing in Steady-State

We begin by solving for the wage from the steady state Euler equation for employment:

\[
W = \frac{MPL}{1+\mu} \left( 1 + \frac{1-x}{\omega(\theta)} - \frac{1}{\omega(\theta) \gamma} \right)
\]

Also, from the Nash Bargaining equation:

\[
W = \phi \frac{MPL}{1+\mu} \left( 1 + \theta \frac{\beta}{\gamma} + \frac{1-x}{\omega(\theta)} \left( 1 - \frac{\beta}{\gamma} \right) \right) + (1 - \phi) \varphi c
\]

In the steady state allocation, in which bond holdings are constant, the budget constraint of the household, combined with the borrowing constraint of entrepreneurs, allows us to express consumption as follows:

\[
c = \beta W n + (1 - \beta) \chi K_T
\]

Combining the steady state Euler equation with the Nash-Bargaining equation and substituting for consumption we get the following labor market clearing expression:

\[
MPL \left( 1 + \frac{1-x}{\omega(\theta)} - \frac{1}{\omega(\theta) \gamma} \right) = \phi \frac{MPL}{1+\mu} \left( 1 + \theta \frac{\beta}{\gamma} + \frac{1-x}{\omega(\theta)} \left( 1 - \frac{\beta}{\gamma} \right) \right) + (1 - \phi) \varphi (\beta W n + (1 - \beta) \chi K_T)
\]

From the Euler equation for tangible capital we get the tangible capital - output ratio:

\[
\frac{K_T}{Y} = \frac{\gamma \omega(\theta)}{1 - \mu \gamma - \gamma (1 - \delta T)}
\]

Finally, using the fact that \( MPL = (1 - \alpha K_T - \alpha K_I) \frac{Y}{T} \), \( L = n - v = \frac{f(\theta) - \theta x}{f(\theta) + 2} \) and \( \omega(\theta) = \frac{f(\theta)}{\theta} \), we can rewrite the labor market clearing expression (after diving both sides by \( Y \) and simplifying) as:

\[
\left( \frac{f(\theta) + x}{f(\theta) - \theta x} \right) \left[ 1 + \frac{\theta}{f(\theta) + 2} \left( 1 - x - 1/\gamma \right) - \phi \left( 1 + \theta \frac{\beta}{\gamma} + \frac{1-x}{\omega(\theta)} \left( 1 - \frac{\beta}{\gamma} \right) \right) \right] =
\]

\[
(1 - \phi) \varphi \beta \left( \frac{f(\theta)}{f(\theta) - \theta x} \right) \left[ 1 + \frac{\theta}{f(\theta) + 2} \left( 1 - x - 1/\gamma \right) \right] + (1 - \phi) \varphi (1 - \beta) \frac{(1+\mu)(\alpha K_T/\gamma)}{1 - \mu \chi - \gamma (1 - \delta T)} \chi
\]

Solving for the labor market equilibrium implies solving for the market tightness \( \theta \). The last equation defines market tightness as a function only of parameters and \( \chi \). Productivity shocks do not appear in this equation, which means that in the steady state the ratio of recruiters to unemployed workers is independent of productivity.\(^{22}\) The high non-linearity of this equation prevents us from getting a closed form solution for \( \theta \) as a function of \( \chi \), but using total derivatives we can show that there is a negative relationship between the two.\(^{23}\)

\(^{22}\) Shimer (2010) shows that this neutrality result for the steady state also carries through to the stochastic simulation of a model with no capital, one type of labor and no financial frictions.

\(^{23}\) The numerical simulation allows us to verify this relationship, by changing the steady state level of \( \chi \) and comparing the results.
7.4 Estimation of Stochastic Processes

We use quarterly data to construct our series of TFP and financial shocks, but given that in our model the stochastic processes have a monthly frequency, we need to transform our quarterly estimates into monthly numbers. In order to do so, we solve the non-linear system of equations associated to the problem of temporal aggregation as follows.  

The monthly auto-regressive process can be represented as:

\[
\hat{X}_t^M = A\hat{X}_{t-1}^M + B\xi_t^M
\]

where \( \hat{X}_t^M = \begin{bmatrix} \hat{z}_t^M \\ \hat{\chi}_t^M \end{bmatrix} \), \( A = \begin{bmatrix} \rho_{zz}^M & \rho_{z\chi}^M \\ \rho_{z\chi}^M & \rho_{\chi\chi}^M \end{bmatrix} \), \( B = \begin{bmatrix} \sigma_{zz}^M & \sigma_{z\chi}^M \\ \sigma_{z\chi}^M & \sigma_{\chi\chi}^M \end{bmatrix} \) and \( \xi_t^M \sim N(0, \sum_{z,\chi}^M = I_2) \). We can re-write it as:

\[
\hat{X}_t^M = A^3\hat{X}_{t-3}^M + B\xi_{t-1}^M + AB\xi_{t-2}^M
\]

and then, re-state it in quarterly terms:

\[
\hat{X}_t^Q = C\hat{X}_{t-1}^Q + D\epsilon_t
\]

where \( \hat{X}_t^Q = \begin{bmatrix} \hat{z}_t^Q \\ \hat{\chi}_t^Q \end{bmatrix} \), \( C = \begin{bmatrix} \rho_{zz}^Q & \rho_{z\chi}^Q \\ \rho_{z\chi}^Q & \rho_{\chi\chi}^Q \end{bmatrix} \), \( B = \begin{bmatrix} \sigma_{zz}^Q & \sigma_{z\chi}^Q \\ \sigma_{z\chi}^Q & \sigma_{\chi\chi}^Q \end{bmatrix} \), \( A^3\hat{X}_{t-3}^M = C\hat{X}_{t-1}^Q \) and \( D\epsilon_t = B\xi_t^M + AB\xi_{t-1}^M + A^2B\xi_{t-2}^M \), with \( \epsilon_t \sim N(0, \sum_{z,\chi}^Q = I_2) \). From our quarterly estimates we get matrices \( C \) and \( D \). Matrix \( A \), governing the persistence of the monthly process can be calculated using \( A = C^{\frac{1}{3}} \). Finally, we can recover matrix \( B \) by solving the following non-linear system:

\[
DD' = BB' + ABB'A' + A^2BB'A'^2
\]

\footnote{For a discussion on this topic see Marcellino (1999).}