Entry, Trade Costs, and International Business Cycles

Roberto N. Fattal Jaef[a,1,*], Jose Ignacio Lopez[b]

[a] *The World Bank. 1818 H st NW mail-stop MC 3-353, Washington DC, 20433*
[b] *HEC Paris. 1 Rue de la Liberation 78351 Jouy en Josas Cedex, France*

Abstract

Are firm entry and fixed exporting costs relevant for understanding the international transmission of business cycles? We revisit this question using a model that includes entry, selection to exporting activity, physical capital accumulation and endogenous labor supply. We determine that once the stochastic process for exogenous productivity is calibrated to consider the endogenous dynamics in TFP created by the number of firms and the time series volatility of entry is calibrated to the data, our model yields minimal departures from the Backus, Kehoe and Kydland (1992) benchmark. The richer model shares all of the successes of the previous model in terms of the volatilities of aggregate quantities, as well as its failures, in terms of replicating patterns of international co-movement and the volatility of international relative prices.

Keywords: International Business Cycle, Extensive Margin, Entry, Fixed Export Costs

1. Introduction

Since the work of Backus, Kehoe, and Kydland (1992), a notably active line of research has attempted to reconcile the two-country version of the neoclassical business cycle model with the observed patterns of international business cycles. This paper focuses on two new mechanisms, firm entry to domestic markets and endogenous export participation decisions, which have received significant attention in the area of international trade but whose quantitative implications for international business cycles are not yet as well understood[2]. Is the consideration of these margins of adjustment important for understanding the international transmission of productivity shocks? Can they help solve any of the long-standing puzzles related to the standard international business

*Corresponding author
Email addresses: rfattaljaef@worldbank.org (Roberto N. Fattal Jaef), lopez@hec.fr (Jose Ignacio Lopez)

[1] The views expressed here are my own, and do not represent those of the World Bank Group or any of its member countries

[2] The literature review section discusses how our paper relates to the works by Ghironi and Melitz (2005) and Alessandria and Choi (2007, 2008), who also study the role of the extensive margin of production and exports for international business cycles

cycle model? To further this discussion, we propose an international business cycle model with entry and fixed exporting costs.

The model is a two-country version of Bilbiie, Ghironi and Meltiz (2012) and extends Ghironi and Melitz (2005) to include physical capital accumulation and endogenous labor supply. There is a single non-tradable final good that can be used for consumption or investment. This good is produced according to a CES production function that uses domestic and foreign intermediate inputs. Producers of intermediate inputs operate a Cobb-Douglas production function combining labor and physical capital. Firm-level productivity in the intermediate sector is composed of an aggregate component, which evolves stochastically over time, and an idiosyncratic component, which is fixed after entry. An infinite mass of potential entrants can become producers after paying a sunk cost of entry denominated in effective units of labor, and incumbents can access the export market at the price of a fixed labor cost. Exit occurs at an exogenous rate that is common across firms.

Our findings suggest that entry and exporting decisions yield minimal departures from the standard two-country model with a representative firm. Although the model has a good fit regarding domestic correlations and volatility of output, investment and hours, it is at odds with the data in terms of international variables: 1) the cross-country correlation of investment displays the opposite sign, 2) the correlation between international relative prices and relative consumption is counterfactually positive (the Backus and Smith (1993) puzzle), 3) consumption is counterfactually more strongly correlated across countries than output (the output-consumption anomaly), and 4) international relative prices are significantly less volatile than in the data. In light of these results, we conclude that entry and exporting decisions cannot help overcome the main failures of the standard model, as they leave its counterfactual predictions largely unaffected.

In reaching this conclusion, we treat the contribution of the number of firms to aggregate output exactly as it has been treated in the National Income and Product Accounts (NIPA) by the Bureau of Economic Analysis before the 2013 comprehensive revision and consistently with the current treatment of these expenditures in other developed countries, which we define as the rest of the world in the model[3]. According to these methodologies, expenditures in firm creation are not considered a tangible form of investment that becomes capitalized into the economy's capital stock but rather as an expense, similar to what occurs with R&D and other forms of intangible investment. This is important for our analysis because it implies that the cyclical behavior of entry

[3]The U.S. BEA provides a detailed explanation of the previous and current treatment of intangible investments in NIPA after the changes in the 2013 comprehensive revision. Other countries, which are part of our composite of countries that we call the "rest of the world", and the "foreign" country in the model have yet to take steps in this direction. EUROSTAT, for instance, is expected to review its national accounts in a similar direction after the adoption of the new European System of National and Regional Accounts (ESA 2010) starting in September 2014 (see press release here: http://epp.eurostat.ec.europa.eu/portal/page/portal/esa_2010/documents/technical_press_briefing_ESA_2010.pdf)

manifests as changes in total factor productivity (TFP) rather than the economy's capital stock. Furthermore, it implies that model-based TFP no longer displays the same time-series properties of the underlying shocks but is rather a mixture of an exogenous and endogenous components[4].

We calibrate the model to take into account this interaction between the number of firms and TFP dynamics when specifying the stochastic properties of exogenous productivity. As standard in the literature, we assume that the exogenous productivity shock evolves according to a bivariate vector auto-regressive process of degree one. Differently from the literature, we use this exogenous shock to generate a model-based TFP that is consistent with the estimated Solow residuals from the data. More specifically, our algorithm starts with a guess of a vector of the persistence, spillover, standard deviation and correlation of innovations for the exogenous productivity process. We solve and simulate the model, estimate the model-based VAR coefficients for the endogenous TFP time series and continue to iterate until the resulting coefficients match the estimates from the data. Calibrated this way, we can work under the assumption that our model is the data-generating process.

To clarify the nature of the divergence between exogenous and endogenous TFP, we provide a decomposition of the endogenous component and study its dynamics after a one-standard-deviation exogenous productivity shock. This analysis shows that the persistence in the response of the number of firms tends to magnify the persistence of the exogenous process, leading the model-based TFP to display a higher first-order coefficient of autocorrelation and a subdued response upon shock. When the positive exogenous productivity shock hits the economy, the number of firms is fixed, but the economy reallocates labor away from production into firm creation. Because the hours worked devoted to entry of new establishments are not accounted for in GDP, this manifests as an initial drag on TFP that reverses as the economy expands the set of active producers. As a result, the stochastic process for exogenous productivity needs to be adjusted to have a lower persistence parameter and a higher standard deviation of innovations to account for the TFP dynamics observed in the data. On this dimension, the entry margin provides the economy with an endogenous mechanism to smooth out the exogenous productivity shock.

Our quantitative analysis is also disciplined by the short-run dynamics of establishment entry observed in the data. We view this as a natural prerequisite to the evaluation of the contribution of this margin to the overall behavior of the economy. To accomplish this goal, we propose a cost function that is linear with respect to effective labor costs in the steady state, as in Melitz (2003), but is increasing and convex in deviations of entry from the steady state along the business cycles. We calibrate this cost function to match the volatility of firm establishment from the Business

[4]In the sensitivity analysis section, we also consider the properties of the model consistent with the 2013 review of the NIPA definitions, which defines that research and development (R&D) expenditure, both private and public, should be considered investment.

Dynamics Statistics so that our conclusions are not driven by either over- or under-stating the cyclical fluctuations of the extensive margin of production.

We conclude our paper with a sensitivity analysis with respect to various aspects of our modeling assumptions and calibration, including our adjustment to the exogenous shocks. Our main conclusion remains robust, but we show that adjusting the exogenous process is not innocuous for the dynamics of the model. For instance, we find that ignoring the adjustment of the stochastic process for exogenous productivity, while re-calibrating entry costs to match the volatility of entry to the data, makes the volatility of aggregate variables significantly lower than in the benchmark model as a result of a lower standard deviation of productivity innovations. Similarly, if we do adjust the parameters that govern the shock process but make entry costs a linear function of effective labor, entry becomes much more volatile than in the data, and the response of other variables is significantly muted. The excess volatility of entry in this specification leads to greater fluctuation in labor, as the entry technology is labor-intensive, while reducing the volatility of investment and thereby exports, imports and the trade balance.

The reminder of this paper is organized as follows. Section 2 discusses related papers in the literature and details how this paper differs from previous work. Section 3 describes the benchmark model. In section 4, we present our quantitative results under our benchmark calibration. Section 5 explores the sensitivity of the results with respect to the adjustment in the stochastic process of productivity, the specification and measurement of entry costs, the denomination of entry and fixed export costs, the elasticity of substitution between domestic and internationally traded goods and the role of physical capital and endogenous labor. The last section presents the conclusions.

2. Related Literature

This paper relates to two lines of research. The first line comprises the family of international business cycle models started by the seminal work of Backus, Kehoe and Kydland (henceforth BKK, 1992), who first showed the failures of the two-country extension of the closed-economy Kydland-Prescott (1982) model. Many papers since then have explored various alternatives to reconcile the standard model with its odd predictions regarding aggregate volatilities, international correlations and international relative prices. An incomplete list of works on this topic is as follows: Baxter and Cruccini (1995); Burstein, Kurz, and Tesar (2008); Kollmann (1996); Heathcote and Perri (2002); Kehoe and Perri (2002); Corsetti, Dedola and Leduc (2008); and Heathcote and Perri (2014). From this literature, our paper inherits the basic structure of the model economy. Moreover, there is a parameterization of our model that delivers a specification similar to the BKK benchmark, namely, an economy with a fixed number of producers and exporters, that we use as a benchmark reference for the evaluation of our results.

The second line of research studies the role of the extensive margin of production and exports in propagating aggregate shocks. In a closed-economy setup, Bilbiie, Ghironi and Melitz (2012) argue that firm entry acts as an important endogenous propagation mechanism of productivity shocks. In the context of a two-country model, Ghironi and Melitz (2005) and Alessandria and Choi (2007 and 2008) discuss the effect of the extensive margin of production and exports on the international transmission of productivity shocks across countries. This paper departs from this research in two important aspects: the mapping of the model to the data and the specification of entry costs.

Unlike previous studies, and in line with the national account framework in developed countries, we follow the practice of not recording expenditures in entry as a form of capitalized investment. We show that once this measurement practice is adopted, the model's endogenous dynamics of the number of firms and the share of labor allocated to entry compound with the exogenous productivity shocks in shaping the dynamics of the model-based TFP. In this regard, this paper contributes along three dimensions: analyzing the direction in which the model transforms the properties of the exogenous productivity shock, suggesting a methodology to re-calibrate the shock so that the model-based TFP can be compared to the estimated Solow residuals in the data and showing that the lack of such adjustment severely worsens the overall fit of the model.

This paper also differentiates from Bilbiie, Ghironi and Melitz (2012) and Ghironi and Melitz (2005) in that it proposes a specification of the entry cost function disciplined by a direct measure of firm entry. We use data for the entry of new establishments from the Business Dynamics Statistics to compare the standard deviation of entry relative to GDP between the model and the data. We show that a linear technology with labor-denominated entry costs significantly over-predicts the volatility of entry. Combined with the above-mentioned practice of not capitalizing expenditures in entry, the excess volatility in this variable does not manifest in the volatilities of investment and GDP. Therefore, a model with linear entry costs exhibits a subdued volatility of output, investment, exports and the trade balance. We overcome this issue by proposing an increasing and convex entry cost function whose degree of convexity is calibrated to target the relative standard deviation of entry to GDP observed in the data.

Unlike Ghironi and Melitz (2005), our model features physical capital accumulation and endogenous labor supply. Beyond the measurement discussion, the addition of these two margins is important for the model to exhibit enough volatility and sufficient propagation of the shocks. We also differ from their study in the choice of some critical parameter values, such as variable trade costs and the elasticity of substitution between domestic and foreign goods. We calibrate the variable trade costs to match the import share observed in the US, which gives us a degree of openness in the steady state that is lower than the 27% import share implied by the calibration in Ghironi and Melitz (2005). Secondly, our specification of a nested CES production function for the final goods allows us to separately match the high elasticity of substitution among varieties within a country and the low elasticity of substitution between domestic and foreign goods simultaneously, following most papers in the literature and in line with the volatility of trade flows at business cycle frequency. Overall, we find that the Ghironi and Melitz (2005) findings are not robust to various modeling choices and do not carry through when the model is mapped to the data in the way we propose.

Relative to Alessandria and Choi (2007), we share the finding that an active extensive margin alone, with a constant total number of firms, yields business cycle predictions that are virtually indistinguishable from the BKK benchmark, although we reach this conclusion using a model with fixed exporting costs and not sunk costs, as they do. On this dimension, this paper contributes by showing that a similar conclusion can be extended to changes in the number of firms when discussing the counterfactual predictions of the standard model.

Lastly, our paper also relates to the work of Alessandria and Choi (2008). They also develop a two-country model of international business cycles with entry of firms to domestic and foreign markets, but with the goal of understanding how the economy's exposure to trade might affect the dynamics of the number of firms. Beyond these differences in scope, our work contributes to their paper in suggesting an alternative way to take the model to the data that is consistent with measurement assumptions about the variables of interest in line with the current practices of most countries' statistical offices.

3. Benchmark Model

Our model economy is composed of a representative household, competitive producers of a final good and a continuum of heterogeneous producers of intermediate goods that operate under monopolistic competition. There are two countries, home and foreign, that can trade intermediate goods. For the benchmark specification, we assume that households can trade a non-contingent bond[5]. Unless otherwise necessary, we restrict our description of the model to domestic agents, omitting the presentation of foreign households and firms for expositional purposes. When required to do so, we distinguish foreign variables with an asterisk superscript.

3.1. Producer of Final Goods

In every period, there exists a set of domestically produced intermediate goods Ω_t and a set of imported varieties $\Omega^*_{x,t}$ that are available for the production of the final good[6]. The final good can be used for consumption or investment and cannot be internationally traded. Exporting is costly and entails the payment of an iceberg cost $\tau > 1$ for each unit of an intermediate good sold abroad. In addition, trade is costly because firms must incur a per-period fixed export cost to gain access to the foreign goods markets. We describe the features of the exporting activity in more detail later

[5] In the online appendix, we also explore two alternative specifications: complete markets and financial autarky.

[6] The superscript indicates the origin of the good, and the subscript x implies that the good is exported. So, for instance, Ω_t is the set of domestically produced intermediate goods available in the home country, while $\Omega^*_{x,t}$ are those goods that are produced abroad and exported to the home economy

in the paper, but for this subsection, it is worth noting that the set $\Omega^*_{x,t}$ changes over time as both home and foreign economies are hit by aggregate productivity shocks. Furthermore, the set Ω_t will also be time-varying because our model features endogenous entry along the business cycle.

Let $z \in \Omega$ be a particular variety. The producer of final goods combine intermediate inputs from both home and foreign according to:

$$Y_t = \left[\left[\int_{z\in\Omega_t} y_t(z)^{\frac{(\theta-1)}{\theta}} dz \right]^{\frac{\theta}{\theta-1}\frac{\rho-1}{\rho}} + \left[\int_{z\in\Omega^*_{x,t}} y^*_{x,t}(z)^{\frac{(\theta-1)}{\theta}} dz \right]^{\frac{\theta}{\theta-1}\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{1}$$

This specification is flexible enough to capture potential different elasticities of substitution within the set of domestic goods, and between domestic and foreign varieties. Here, θ denotes the former and ρ the latter. The demand for the domestic variety z is denoted by $y_t(z)$, while imports of the home country (exports of the foreign country) are referred as $y^*_{x,t}(z)$. An identical technology is assumed for the foreign country.

Let P_t be the price of the home final good, taken as given by the competitive producer of such good, and let $p(z)$ and $p^*_x(z)$ denote the price of a domestic and a foreign variety z, respectively. Then, the problem of the producer of final goods is given by:

$$\max_{y_t(z), y^*_{x,t}(z)} P_t Y_t - \int_{z\in\Omega_t} p_t(z) y_t(z) dz - \int_{z\in\Omega^*_{x,t}} p^*_{x,t}(z) y^*_{x,t}(z) dz$$

subject to the technology in (1). The solution to this problem gives the following demand and aggregate price functions:

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{2}$$

$$y^*_{x,t}(z) = \left(\frac{p^*_{x,t}(z)}{P_t}\right)^{-\theta} \left(\frac{P^*_{X,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{3}$$

$$P_t = \left[P_{D,t}^{1-\rho} + P_{X,t}^{*1-\rho} \right]^{\frac{1}{1-\rho}} \tag{4}$$

where we have defined $P_{D,t} \equiv \left[\int_{z\in\Omega_t} N_{D,t} p(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$ and $P^*_{X,t} \equiv \left[\int_{z\in\Omega^*_{x,t}} N^*_{X,t} p^*_x(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$. The aggregate price level in the home country is thus a CES aggregator of two other CES aggregates of domestic and imported varieties. The demand functions depend negatively on the variety's price relative to the aggregate price level, $\frac{p_t(z)}{P_t}$, with the price elasticity given by θ. Moreover, so long $\theta \neq \rho$, the demand for domestic varieties will also be a function of the aggregate price of domestic varieties relative to the aggregate price index in the economy, $\frac{P_{D,t}}{P_t}$. An equivalent relationship holds for home country's imports. Finally, since the final good will be either consumed or invested, in equilibrium we will have $Y_t = C_t + I_t$.

3.2. Producers of Intermediate Goods

We start the analysis of the intermediate goods sector describing the optimization problem of a preexisting producer and then characterizing entry decisions.

3.2.1. Pricing and the Decision to Export

Firms are differentiated by their productivity z, so there is no loss of generality in indexing varieties with productivities, as we have done above[7]. At any point in time there exists a mass $N_{D,t}$ of firms that are actively producing and selling in domestic markets. These firms act under monopolistic competition, so they take the demand functions for their variety as given. We assume that production is done with a Cobb Douglas production function with identical factor shares across varieties:

$$Y_t(z) = Z_t z \left[k_t(z)\right]^{\alpha} \left[l_t(z)\right]^{1-\alpha}$$

where $k_t(z)$ and $l_t(z)$ represent the demand for capital stock and labor units from a producer with productivity z. The notation extends naturally to the foreign country with the inclusion of the star superscript. Heterogeneity is reflected in differences in the TFP component of each firm z, which we assumed to be fixed. All firms are subject to a common aggregate productivity factor Z_t, which evolves stochastically in our model .

Conditional on entry, firms can decide to engage in exporting activity. We assume that there exists a per period fixed export cost $\hat{f}_x$, measured in effective units of labor, that leads only a subset of firms to find exporting profitable.

Prices are expressed in units of the final good at destination. Then, $\rho_{x,t}(z) = \frac{p_{x,t}(z)}{P_t^*}$ is the price set by a domestic producer for its sales in the foreign market, while the real value of domestic varieties sold in the home country is $\rho_t(z) = \frac{p_t(z)}{P_t}$. Firms can aggregate domestic and foreign profits flows converting the latter into units of the home final good through the real exchange rate $Q_t = \frac{\varepsilon P_t^*}{P_t}$.

Given these definitions, a domestic intermediate producer maximizes (5) subject to (6), (7) and (8).

[7]Although we refer to the production units in our model as firms, we acknowledge that these do not correspond exactly to what firms are in reality, where a firm may have different establishments with different productivities. Thus, we are implicitly imposing in the model that a firm is equivalent to an establishment. Furthermore, it is also arguable that a variety is actually a product; therefore the model can be re-interpreted as a description of product innovation along the cycle

$$\max \rho_t(z) y_t(z) + x(t) Q_t \rho_{x,t}(z) y_{x,t}(z) - w_t l_t(z) - r_t^k k_t(z) - x(t) \frac{w_t}{Z_t} \hat{f}_x \tag{5}$$

$$y_t(z) + x(t) \tau y_{x,t}(z) = Z_t z \left[k_t(z)\right]^{\alpha} \left[l_t(z)\right]^{1-\alpha} \tag{6}$$

$$y_t(z) = \left(\frac{p_t(z)}{P_t}\right)^{-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \tag{7}$$

$$y_{x,t}(z) = \left(\frac{p_{x,t}(z)}{P_t^*}\right)^{-\theta} \left(\frac{P_{X,t}}{P_t^*}\right)^{\theta-\rho} Y_t^* \tag{8}$$

The term $x(t)$ is an indicator function that captures the export status of the firm, being equal to 1 when it exports and 0 otherwise. Letting $MC_{p,t}(z)$[8] denote the marginal cost of production, the optimal pricing rule is defined by:

$$\rho_t(z) = \frac{\theta}{\theta - 1} MC_{p,t}(z) \tag{9}$$

$$\rho_{x,t}(z) = Q_t^{-1} \frac{\theta}{\theta - 1} \tau MC_{p,t}(z) = Q_t^{-1} \tau \rho_t(z) \tag{10}$$

which is the familiar result for problems with monopolistic competition and aggregate CES technology. Since marginal costs are a negative function of the firm-specific productivity term z, more productive firms set lower prices. The choice of factors of production is determined by:

$$r_t^k = MC_{p,t}(z) \alpha \frac{Y_t(z)}{k_t(z)} \tag{11}$$

$$w_t = MC_{p,t}(z)(1-\alpha) \frac{Y_t(z)}{l_t(z)} \tag{12}$$

Under monopolistic competition, prices no longer equals marginal costs, implying that factor demands are different from those in the efficient allocation[9].

Profits from domestic sales, $d_{D,t}(z)$, are:

$$d_{D,t} = \frac{1}{\theta} \left[\rho_t(z)\right]^{1-\theta} \left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t$$

[8] With Cobb Douglas production function and labor share $1 - \alpha$, the marginal cost is of the form $\frac{r^{\alpha} w^{1-\alpha}}{Zz} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha}$, where r is the rental rate of capital stock in units of the home final good, and w is the real wage.

[9] Explicit formulas for the demand of production factors are derived in the online appendix.

while profits from export sales[10], $d_x(z)$, are described by:

$$d_{x,t} = \left\{ \frac{Q_t}{\theta} \left[\rho_{x,t}(z)\right]^{1-\theta} \left(\frac{P_{X,t}}{P_t^*}\right)^{\theta-\rho} Y_t^* - \frac{w_t}{Z_t}\hat{f}_x \right\}$$

if firm z exports, and zero otherwise.

It follows that more productive firms set lower prices and earn higher profits. This is a feature of CES demand functions, which exhibit elasticities with respect to price that are greater than one in absolute value ($\theta > 1$). Since profits are increasing in productivity and the fixed export cost imposes a non-convexity that leads to a decreasing average total cost curve, only the most productive firms export. In particular, since the relationship between profits and productivity is monotonic, we can characterize the export choice with a cut-off rule of the form:

$$z_{x,t} = \inf\left\{z : d_{x,t}(z) > 0\right\} \tag{13}$$

where all firms whose productivity is greater than or equal to the cutoff productivity $z_{x,t}$ decide to export. Provided the lowest productivity firm is sufficiently low to ensure that $z_{x,t}$ is in the interior of the support of the distribution, there always exists a subset of firms that decide not to export, endogenously determining a non-traded sector in the economy. Importantly, the size of this sector will differ across countries and change over time in response to aggregate technology shocks.

Finally, we define fixed exporting costs using the following specification:

$$\hat{f}_x = f_x + \gamma_x \left[exp\left(z_{x,t} - z_x\right) - 1\right]$$

where z_x refers to the steady-state value of the export threshold. For our benchmark specification we set $\gamma_x = 0$, so $\hat{f}_x$ is constant along the cycle and fixed exporting costs are only affected by changes in the effective cost of labor. When this parameter is set to a positive number, fixed exporting costs also adjust to responses in cyclical changes of the export threshold. Effectively, a parameterization of the model with a very large γ_x allows us to shut down the extensive margin of exports and consider an economy in which there is a constant fraction of exporting firms, without altering the steady-state of the model.

3.2.2. Entry, Exit and Ownership of Firms

Entry is modeled following Melitz (2003). There is an infinite pool of forward looking potential entrants that consider paying a sunk entry cost $\hat{f}_e$ of effective units of labor to get a productivity draw z from a common known distribution $G(z)$, identical in both countries, with support $[z_{\min,}\infty)$. We assume no fixed costs of operation in the domestic markets; hence, all firms in the support have

[10] We can split domestic from export profits because firms produce under constant returns to scale.

positive profits. The idiosyncratic productivity is constant over the lifetime of the firm, which can be interrupted at any time with exogenous probability δ. These assumptions combined imply that there is a fixed *ex-post* distribution of productivities that is identical to the *ex-ante* distribution $G(z)$, keeping the aggregation of the model tractable over the business cycle[11].

Because productivities are drawn after payment of the entry cost, prospective entrants consider the average value of a firm in making their entry decisions. This is determined by the expected present value of average total profits and must equal the entry cost in equilibrium:

$$\hat{f}_e \frac{w_t}{Z_t} = \tilde{v}_t$$

Here, $\tilde{v}_t = E_t \sum_{\tau=t+1}^{\infty} [\beta(1-\delta)]^{\tau-t} \frac{U_c(c_\tau, l_\tau)}{U_c(c_t, l_t)} \tilde{d}_\tau$ is the present value of the expected stream of average profits $\tilde{d}_t$, discounted at the stochastic discount factor augmented to account for the exogenous probability of exit δ. Note that this representation of entry costs implies that the entry technology is subject to productivity shocks. This assumption, as discussed by Bilbiie, Ghironi and Melitz (2012), is consistent with the notion that productivity shocks are truly aggregate as they affect both the production and the entry technology. In addition, as we discuss in the last section of the paper, this feature is key to generate in the model a pro-cyclical entry.

To give persistence to the evolution of the mass of domestic producers, we assume a one period lag for the new entrants before they become operative. In this way, the mass of domestic producers is a state variable in the model, that evolves according to:

$$N_{D,t} = (1-\delta)(N_{D,t-1} + N_{e,t-1}) \tag{14}$$

As in Ghironi and Melitz (2005), firms are owned by a mutual fund that pays dividends equal to the average total profits of active firms, $\tilde{d}_t$. Households can trade shares of the mutual fund in the stock market at a price $\tilde{v}_t$. The definition of the average value of the firm as the present value of the expected stream of average profits follows from the first order condition of the representative household's maximization problem.

Analogous to our previous assumption regarding fixed exporting costs, we define entry costs using the following expression:

$$\hat{f}_e = f_e + \gamma_e [\exp(N_{e,t} - N_e) - 1]$$

The parameter γ_e governs directly the volatility of entry. A very large value of γ_e reduces entry to the point that effectively there is no net entry along the cycle. This specification of entry and

[11]So long our focus is on the aggregate implications of entry and fixed exporting costs, we do not view these assumptions as excessively restrictive. They would certainly be much more limiting if we were interested in matching firm-level moments of the data along the business cycle (such as firm growth and size).

fixed costs nests two limiting cases of interest: 1) setting γ_e and γ_x to be sufficiently large, it would be as considering an economy with a constant number of firms and a fixed fraction of exporters, resembling the BKK specification[12], 2) setting γ_e sufficiently large and $\gamma_x = 0$, we would be considering an economy where the number of firms is constant, differing from BKK only in that there is an extensive margin of trade. This is an interesting special case, since it is close to the environment study in Alessandria and Choi (2007)

Although we have chosen this representation of entry costs mainly because it allows us to analyze the role of entry in the model, it is not completely arbitrary. Other papers that deal with entry allow too for some curvature of entry costs, which can be understood as an expression of diminishing quality in managerial ability or congestion effects at firm creation.

3.2.3. Aggregation

As mentioned before, the firms' idiosyncratic productivities are distributed according to $G(z)$, and there exists a cut-off productivity separating the set of exporting and non-exporting firms. The mass of exporting firms is then:

$$N_{x,t} = [1 - G(z_{x,t})]\, N_{D,t}$$

which pins down the share exporting firms directly from the assumed distribution of productivities and the cut-off.

It can be shown that all micro-level variables (individual prices, profits, etc) depend on the idiosyncratic productivity in the form $z^{\theta-1}$. For instance, the profits from domestic sales, having replaced prices from the optimal rule, can be written as:

$$d_{D,t}(z) = \frac{1}{\theta}\left[\frac{r_t^{\alpha} w_t^{1-\alpha}}{Z_t}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\right]^{1-\theta}\left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} z^{\theta-1} Y_t$$

From here, it follows that *average* domestic profits are given by:

$$\tilde{d}_{D,t} = \frac{1}{\theta}\left[\frac{r_t^{\alpha} w_t^{1-\alpha}}{Z_t}\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\right]^{1-\theta}\left(\frac{P_{D,t}}{P_t}\right)^{\theta-\rho} Y_t \int_{z_{\min}} z^{\theta-1} dG(z) \tag{15}$$

Following Melitz (2003), we can construct a pair of statistics $\tilde{z}_D$ and $\tilde{z}_{x,t}$ (similarly for the foreign country) that summarize the entire distribution of productivities of all firms and of the subset of exporting firms in the economy:

[12] This model differs to the original BKK as it features monopolistic competition in the intermediate goods sector and that although net entry is zero, there is entry and exit in the model, hence there is a fraction of labor used to pay sunk and fixed costs.

$$
\begin{aligned}
\tilde{z}_D &\equiv \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}} \\
\tilde{z}_{x,t} &\equiv \left[\frac{1}{1-G(z_{x,t})}\int_{z_{x,t}} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}
\end{aligned}
$$

The average productivity of domestic firms, $\tilde{z}_D$, is independent of time as a consequence of the assumption of no fixed costs of operation, which implies that all firms produce. On the other hand, the average productivity of exporters changes over time in response to aggregate productivity shocks.

The particular form in which micro-level variables depend on productivity implies that we can refer to average values of the variable with its value at the average firm. Then, we can write, for instance, average domestic profits $\tilde{d}_{D,t}$ as the profits of the average firm $d_{D,t}(\tilde{z}_D)$. The results extends to average exporting profits $\tilde{d}_{x,t} \equiv d_{x,t}(\tilde{z}_{x,t})$, average relative prices $\tilde{\rho}_t \equiv \rho_t(\tilde{z}_D)$ and $\tilde{\rho}_{x,t} \equiv \rho_{x,t}(\tilde{z}_{x,t})$; and average total profits $\tilde{d}_t = \tilde{d}_{D,t} + [1 - G(z_{x,t})]\, \tilde{d}_{x,t}$.

Finally, having characterized the microeconomics underpinnings of the model, we can refine the notation we used in the description of the final goods sector in order to characterize the consumption baskets, Ω_t and $\Omega^*_{x,t}$. The former is composed of all the firms in the economy and has mass equal to $N_{D,t}$, while the latter consists of the subset of firms with $z \in [z^*_{x,t}, \infty)$ and has mass equal to $N^*_{x,t}$. The aggregate price indexes, using the aggregation properties of the model, can therefore be written as:

$$
P_{D,t} \equiv N_{D,t}^{\frac{1}{1-\theta}} \tilde{p}_t \tag{16}
$$

$$
P^*_{x,t} \equiv N_{x,t}^{\frac{1}{1-\theta}} \tilde{p^*}_{x,t} \tag{17}
$$

In summary, two key features of the model keep the aggregation simple and tractable at business cycle frequency: an exogenous exit rate and the absence of idiosyncratic productivity shocks. As a result of these two assumptions, the productivity distribution of firms is equal to the ex ante distribution regardless of the realization of the aggregate shocks. In addition, the shape of the CES demand function allows for the representation of aggregate variables in terms of sufficient statistics of the distribution.

3.3. Household's Problem

Each country is populated by a unit mass of households that choose consumption, investment in physical capital, investment in financial assets and hours worked to maximize lifetime utility according to:

$$\max E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^{\mu} \left(1 - L_t\right)^{1-\mu} \right)}{1-\gamma} \right]^{1-\gamma}$$

where C_t denotes consumption, L_t represents ours worked, the parameter $\beta \in (0,1)$ is the discount factor, γ is the coefficient of relative risk aversion and μ the share of consumption in utility.

The components of the budget constraint depend on the degree of completeness in international financial markets. Our benchmark environment is one in which financial markets are exogenously incomplete, allowing households to trade only non-contingent bonds across borders[13]. In addition, households can trade shares on the mutual funds that has property of all firms in the economy, although we assume stock markets are only open to agents of a given country.

Letting φ_t be the shares on the domestic mutual fund held by domestic household, the total amount of resources spent to accumulate shares is $\varphi_{t+1}\left(N_{D,t} + N_{e,t}\right)\tilde{v}_t$, where $\left(N_{D,t} + N_{e,t}\right)$ is the number of firms owned by the mutual fund and $\tilde{v}_t$ its average value in units of the home final good. For share holdings of φ_{t+1} in the next period, the household receives dividends $\tilde{d}_{t+1}$ from a fraction $(1-\delta)$ of the $\left(N_{D,t} + N_{e,t}\right)$ firms in the mutual fund, and has the option to sell his shares at a value of $\tilde{v}_{t+1}N_{D,t+1}$. Regarding the accumulation of domestic and foreign bonds, we introduce adjustment costs in the household's bond holdings in order to recover the stationarity of the model, as adopted by Ghironi and Melitz (2005) and discuss in detail by Schmitt-Grohe and Uribe (2003). We denote home country's agent holding of domestic and foreign bonds with B_t and $B_{*,t}$, which earn a risk free rate of r_t^B and $r_t^{B,*}$. Positions in the foreign country are defined symmetrically, being identified with a $*$ superscript.

The budget constraint in the home country is:

$$\begin{aligned} & B_{t+1} + Q_t B_{*,t+1} + \frac{\eta}{2}\left(B_{t+1}^2 + QB_{*,t+1}^2\right) + C_t + K_{t+1} + \varphi_{t+1}\tilde{v}_t\left(N_{D,t} + N_{e,t}\right) \\ = \quad & (1 - \delta_k + r_t^k)K_t + (\tilde{d}_t + \tilde{v}_t)\varphi_t N_{D,t} + w_t L_t + \left(1 + r_t^B\right)B_t + \left(1 + r_t^{B,*}\right)Q_t B_{*,t} + T \end{aligned} \tag{18}$$

where units are already in terms of the final good, and T stands for the lump sum rebate of the collected resources from the adjustment costs of bond holdings.

3.4. Definition of Competitive Equilibrium

A *competitive equilibrium* in this economy is (the same elements apply to the foreign country): *a)* a set of consumption, labor, investment in physical capital and investment in financial asset choices, *b)* a set of price rules, factor demands and export decisions, *c*) demand functions for intermediate

[13]Since the dynamics of both micro and macro level variables in the model could be changed by the degree of risk sharing across countries, we devote a section in the online appendix to study the role of this assumption, considering two additional cases: financial autarky and complete markets

goods, *d)* a mass of new entrants N_e, *e)* a cutoff productivity z_x, *f)* a set of iceberg, entry and fixed export costs and *g)* a vector of asset prices, wages and interest rates such that: 1) given *g and d*, *a* solves household's maximization problem; 2) given *c, e, f* and *g*; *b* solves the problem of the producers of intermediate goods and *e* satisfies the zero cutoff profit condition, 3) given *f* and *g*, *d* satisfies the free entry condition; and 4) markets clear.

The wage rate and the real return to capital adjust so that supply equals demand in the labor and capital rental markets:

$$K_t = K_{p,t}$$

$$L_t = L_{p,t} + L_{e,t} + L_{x,t}$$

Here, $L_{p,t}$, $L_{e,t}$, and $L_{x,t}$ stand for aggregate labor demands in the production of intermediate goods, entry, and fixed exporting costs, respectively, and they are defined by the following expressions:

$$L_{p,t} = \frac{N_{D,t}\left[\frac{r_t^k(1-\alpha)}{w_t\alpha}\right]^{\alpha}\left[\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\frac{\theta}{\theta-1}\right]^{-\theta}Z_t^{\theta-1}}{\left[\left(\frac{P_t}{P_{D,t}}\right)^{\rho-\theta}Y_t\tilde{z}_D^{\theta-1}+\frac{N_{x,t}}{N_{D,t}}\tau^{1-\theta}Q_t^{\theta}\left(\frac{P_t^*}{P_{X,t}}\right)^{\rho-\theta}Y_t^*\tilde{z}_{x,t}^{\theta-1}\right]} \tag{19}$$

$$L_{e,t} = N_{e,t}\hat{f}_{e,t} \tag{20}$$

$$L_{x,t} = N_{x,t}\hat{f}_{x,t} \tag{21}$$

Capital demand, in turn, is defined as:

$$K_t = \frac{N_{D,t}\left[\frac{w_t\alpha}{r_t^k(1-\alpha)}\right]^{1-\alpha}\left[\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}A\frac{\theta}{\theta-1}\right]^{-\theta}Z_t^{\theta-1}}{\left[\left(\frac{P_t}{P_{D,t}}\right)^{\rho-\theta}Y_t\tilde{z}_D^{\theta-1}+\frac{N_{x,t}}{N_{D,t}}\tau^{1-\theta}Q_t^{\theta}\left(\frac{P_t^*}{P_{X,t}}\right)^{\rho-\theta}Y_t^*\tilde{z}_{x,t}^{\theta-1}\right]}$$

Notice that capital is only demanded for the production of intermediate inputs, while labor is also used for the creation of new firms and for allowing existing firms to export.

The risk free rates of return in equilibrium requires that domestic and foreign bond markets clear, $B_t + B_{*,t} = 0$ and $B_t^* + B_{*,t}^* = 0$; and equilibrium asset prices must adjust so that $\varphi_{t+1} = \varphi_{t+1}^* = 1$. The evolution of net foreign assets in equilibrium is given by:

$$B_{t+1} + Q_tB_{*,t+1} = (1 + r_t^B)B_t + Q_t(1 + r_t^{*,B})B_{*,t} + (w_tL_t + r_t^kK_t + N_{D,t}\tilde{d}_t - N_{e,t}\tilde{v}_t - C_t - I_t)$$

$$B_{t+1}^* + Q_tB_{*,t+1}^* = (1 + r_t^B)B_t^* + Q_t(1 + r_t^{*,B})B_{*,t}^* + Q(w_t^*L_t^* + r_t^{k,*}K_t^* + N_{D,t}^*\tilde{d}_t^* - N_{e,t}^*\tilde{v}_t^* - C_t^* - I_t^*)$$

where the foreign country's condition has been expressed in units of the final good in the home economy using the real exchange rate, Q.

3.5. Measurement

Before discussing our quantitative results, it is useful to discuss some issues regarding the mapping between the model and the data. One important aspect to consider is whether sunk entry costs and fixed exporting costs are capitalized into the economy's gross private domestic investment or are expensed, namely, treated as an intangible investment that is not measured in the economy's value added. The new revision of the NIPA accounts following the System of National Accounts 2008 have changed the accounting definition regarding Research and Development (R&D) expenditures. Under the new definition, R&D expenditures should be accounted as investment. Prior to this revision, R&D was not capitalized, hence entry costs shouldn't be included in investment when comparing the model with the current time series describing international business cycles[14].

Consistently with this, when we report business cycle statistics and impulse response functions for investment and GDP, we refer to the following expressions:

$$I_t = K_{t+1} - (1 - \delta)K_t$$

$$GDP_t = Y_t + TB_t$$

where,

$$TB_t = \int_{z\in\Omega_{x,t}} p_{x,t}(z)y_{x,t}(z)dz - \int_{z\in\Omega^*_{x,t}} p^*_{x,t}(z)y^*_{x,t}(z)dz$$

This approach to measuring output implies that when discussing TFP shocks and their stochastic properties, we need to take into account the endogenous dynamics of the number of firms in the model. We discuss in detail in the next section the required adjustment when calibrating the TFP process that is consistent with this measurement assumption regarding entry and fixed exporting costs.

Another issue when mapping the model to the data is related to the comparison between the model's price indices and their data counterpart. A well-established property of CES production functions, as the one we adopt here for the production of final goods, is that they exhibit "love for variety". As the economy experiences entry both to domestic and foreign markets, the aggregate price index fluctuates even if average prices of already existing varieties remained constant. There is empirical evidence suggesting that this love for variety effect go mostly unmeasured in consumer price indices, as documented in Broda and Weinstein (2006). One feasible adjustment is to construct price indices that resemble the procedure use by statistical offices, which only account for a fix set

[14]Other statistics agencies are expected to include in the near future R&D expenditures as part of aggregate investment. European Statistical agencies are expected to adopt this new standard in September 2014

of already existing varieties. In our model, prices of existing varieties only depend on marginal costs and hence on a combination of wages and the rate of return of physical capital. Using this definition we can compute an alternative definition to the theoretical price index and use it to construct a real exchange rate that is not directly affected by the changing number of varieties and is only a function of the relative ratio of wages and return on physical capital. We label this alternative way to measure the real exchange rate as the Terms of Efficiency (TOE):

$$TOE = \left(\frac{r_t^*}{r_t}\right)^{\alpha} \left(\frac{w_t^*}{w_t}\right)^{1-\alpha} \frac{Z_t}{Z_t^*} \tag{22}$$

Nonetheless, this alternative specification does not solve the issue of how to compute real variables in the model. Without a unit of account, there is nothing in the model that pins down the price of the final good. Hence, all variables, including wages, are expressed relative to the final good price.

To circumvent this issue, other papers such as Ghironi and Meltiz (2005) and Alessandria and Choi (2007) construct a price index where average prices are weighted by the total number of varieties. This adjustment continues to be odds with price indices measured in the data as it includes updates to the share of domestic and exported goods every period, while updates in the consumption basket are done sporadically by statistical agencies.

For our quantitative exercise we use the real variables as defined in the model, relative to the price of the final consumption good. When discussing the real exchange rate we present the two versions: the theoretical real exchange rate, which includes the variety effect, and a real exchange rate that is only based on the terms of efficiency and ignores any direct effect on the number of varieties on prices. The appendix includes the derivation of the real exchange rate under these two specifications.

4. Quantitative Analysis

4.1. Calibration of Model Parameters

We choose values for the set of relevant parameters to match features of the US economy at a quarterly frequency. A summary of the calibrated parameters of the benchmark model is presented on Table 1. We follow the standard choices in the literature regarding a discount factor of households (β) of 0.99 and an inter-temporal elasticity of substitution (γ) of 2 and set μ, which governs the share of consumption in the household's utility, to target a steady-state level of 1/3 for hours worked. We also follow the usual choice for the depreciation rate of the capital stock (δ_K) of 0.025 and pick α equal to 0.36 to obtain a capital income share of 0.3[15].

[15] In our model, α is not exactly the capital income share given the presence of profits

Iceberg costs (τ) are set to match an import share of 15%. We choose the fixed cost of exporting parameter, f_x, to match a steady-state fraction of exporting firms of 21%, in line with the empirical findings for the US of Bernard and Jensen. (2004). As long as we do not target a specific number for the mass of domestic firms in the steady state, we have one degree of freedom for choosing f_e, so we normalize it to one[16]. Following Bernard et al. (2003), we choose the Pareto shape parameter κ to be 5.6 to reproduce in the model the standard deviation of log sales of 1.67 in US plants, which is a typical target in models with heterogeneous producers.

We calibrate the elasticity of substitution among domestic varieties (θ) to a value of 6, which is standard in the international literature and consistent with the elasticity estimates by Broda and Weinstein (2006) and implies a mark-up of 20%. We set ρ to 1.5, as in standard international models and in line with empirical evidence of the elasticity of substitution between home and foreign goods at business cycle frequencies. In the sensitivity analysis, we discuss how the results of the baseline model change when we set both θ and ρ to 3.8, which is a calibration consistent with higher mark-ups and a larger Armington elasticity, as in Ghironi and Melitz (2005).

The death parameter (δ) is set in our benchmark specification to 0.025, which is consistent with an exogenous destruction rate of 10% in annual terms, as in other papers with entry, such as Restuccia and Rogerson (2008), Ghironi and Melitz (2005) and Bilbiie, Ghironi and Melitz (2012). The parameter value targets the average exit rate for the total number of establishments in the Business Dynamics Statistics in the U.S.

The parameter for the cost of adjustment in international bond holding (η) is set to a small number to introduce stationarity in asset holding.

For our benchmark specification, we choose the curvature parameter governing entry costs, γ_e, to match the volatility of entry relative to output. For the parameter governing the curvature of fixed exporting costs, γ_x, we set a value of zero for the benchmark model but consider an alternative specification in the sensitivity analysis section, where both γ_e and γ_x are set to sufficiently high numbers to make the mass of producers and the share-exporting firms effectively constant along the business cycle.

4.2. Calibration of Stochastic Processes

Regarding the specification of the exogenous stochastic processes for aggregate productivities, the calibration exercise needs to take into account that the model delivers endogenous TFP series, for both home and foreign countries, that may not coincide with the estimated Solow residuals as measured in the data. Changes in the number of productive and exporting firms affect the TFP dynamics of the model. Therefore, we must adopt a specification of the stochastic process for

[16]In the appendix, we show that only the ratio f_x/f_e matters for the calibration of the steady-state share of exporting firms.

exogenous aggregate productivities, Z_t and Z_t^*, that, when fed into the model, delivers a model-based measured TFP process consistent with that estimated from the data.

Before explaining the details of the calibration exercise, let us consider the following expression for GDP in the symmetric steady state:

$$Y = N_D^{\frac{1}{\theta-1}} Z \left(\tilde{z}_D^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right)^{\frac{1}{\rho-1}} K^{\alpha} L_p^{1-\alpha} \tag{23}$$

The aggregate output inherits the Cobb-Douglas functional form from intermediate producers' technology, with capital and labor as inputs, and an efficiency term that depends on the exogenous productivity Z, the number of firms N_D, the share of exporting firms N_X and the average productivity of exporters $\tilde{z}_x$. The last three terms are equilibrium objects of the model.

Aggregate output in the model is a function of the amount of productive labor, rather than the total working hours, which also include entry and fixed costs of exporting, which are labor-intensive. However, when estimating the Solow residuals from the data, there is no such distinction, and it is the entire labor in the economy that is taken into account in the input mix. Thus, taking also into account this difference, we define measured TFP in the model as follows:

$$TFP = N_D^{\frac{1}{\theta-1}} Z \left(\tilde{z}_D^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right)^{\frac{1}{\rho-1}} \frac{L_p^{1-\alpha}}{L^{1-\alpha}} \tag{24}$$

This expression, in addition to capturing the equilibrium objects mentioned above, also reflects the cyclical variation in the shares of labor allocated into the production of intermediate goods. From this expression, the elements of the model that drive the difference between the model's TFP and the exogenous Z process can be observed.

The standard calibration for cross-country TFP processes in the international business cycle literature, despite featuring some variation, exhibits fairly persistent processes, with autocorrelation coefficients between 0.9 and 0.99, cross-country correlation of innovations between 0.25 and 0.29 and no spillovers. For our benchmark exercise, we adopt the following TFP specification as our calibration target:

$$\begin{bmatrix} TFP_t \\ TFP_t^* \end{bmatrix} = \begin{bmatrix} .95 & .00 \\ .00 & .95 \end{bmatrix} \begin{bmatrix} TFP_{t-1} \\ TFP_{t-1}^* \end{bmatrix} + \begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} \tag{25}$$

with standard deviations σ_{ψ} and correlation of the innovations $\sigma_{\psi^*\psi}$ given by .007 and .29, respectively.

We define the parameter values of the exogenous stochastic processes, Z_t and Z_t^*, using the following notation:

$$\begin{bmatrix} Z_t \\ Z_t^* \end{bmatrix} = \begin{bmatrix} \rho_z & .00 \\ .00 & \rho_z \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \tilde{v}_t \\ \tilde{v}_t^* \end{bmatrix}$$

with $\tilde{v}_t$ and $\tilde{v}_t^*$ having standard deviation σ_v and correlation σ_{v^*v}.

We start with a guess for $\{\rho_z, \sigma_v, \sigma_{v^*v}\}$ and then log-linearize, simulate the model for a large number of periods and estimate a bivariate VAR using the simulated TFP series in the same way the data are treated[17]. We continue iteratively with this procedure until the model-based estimation of the TFP processes exactly matches the autocorrelation, standard deviation and correlation of shocks in the process given by (25). Finally, note that the outcome of this procedure is a function of other parameters of the model that affect the endogenous elements that show up in the model's TFP. Therefore, we also need to adjust some of these parameters, more specifically γ_e, so that we jointly match the volatility of entry and the stochastic properties of the model's TFP.

The resulting vector of estimates for Z_t and Z_t^* that make the model's TFP match the data is as follows: $\{\rho_z = 0.9, \sigma_v = 0.01, \sigma_{v^*v} = 0.105\}$. Both the persistence parameter and the correlation across innovations are lower than those for the TFP process, while the standard deviation is higher. This suggests that the endogenous forces in the model introduce persistence into TFP dynamics, reduce the volatility of the exogenous component and magnify the correlation of shocks.

Figure 1 illustrates the impulse response function for the model-based TFP in response to an exogenous productivity shock in the home country. We break down such dynamics into the dynamics of the following components[18]:

$$TFP^1 = N_D^{\frac{1}{\rho-1}} \tag{26}$$

$$TFP^2 = \frac{L_p^{1-\alpha}}{L^{1-\alpha}} \tag{27}$$

$$TFP^3 = \left(\tilde{z}_D^{\rho-1} + \tau^{1-\rho} \left(\frac{N_x}{N_D} \right)^{\frac{1-\rho}{1-\theta}} \tilde{z}_x^{\rho-1} \right)^{\frac{1}{\rho-1}} \tag{28}$$

We propose this breakdown to understand how the endogenous components of TFP respond to the exogenous component. The circle-filled line in Figure 1 reproduces the dynamics of the exogenous component, Z_t, in response to a one-standard-deviation positive shock. The square-filled line traces the model's TFP as defined by equation 24, while the other lines track the behavior of each of the three endogenous components defined in equations 26, 27, and 28.

Figure 1 clarifies why the TFP in the model is less volatile than the aggregate productivity.

[17] Solow residuals in the data are expressed as percentage deviations from a linear trend. We apply the same definition and use a linear trend for this exercise.

[18] It is important to note that these three terms, in conjunction with Z, determine the total factor productivity exactly in the model's steady state but not entirely during transitional dynamics, where the trade balance and foreign country's variables (which do not show up in the steady-state formula because of symmetry) come into play.

After the economy is hit by a positive aggregate shock, the share of labor and capital allocated into production of goods declines relative to the total, reflecting the economy's incentives to increase entry. The increase in entry eventually translates into a higher number of firms, magnifying the economy's total factor productivity, but this does not occur until a few quarters after the shock. Along this dimension, entry allows the economy to smooth out the productivity shock. The TFP1, TFP2 and TFP3 lines show that there is almost no action coming from the cyclical behavior in the share and average productivity of exporters, with most of the propagation coming from the number of firms and the relative allocation of labor.

4.3. Business Cycle Statistics

We report in Table 3 the main business cycle statistics of our benchmark model and compare them with quarterly statistics for the US economy and three alternative specifications[19]. For the No Entry specification, we prevent cyclical changes in the number of entrants by choosing a large value for the adjustment parameter γ_e. This specification, although different in some dimensions, resembles the model discussed by Alessandria and Choi (2007), in which there are no cyclical changes in the mass of firms and the only extensive margin at play is given by the changing fraction of exporting firms. Alternatively, the No Export model exhibits a constant share of exporting firms along the cycle as a result of a large value for γ_x. The No Extensive Margin model has both adjustment parameters set to a sufficiently large value such that both the mass of firms and the fraction of exporting firms remain unchanged, as in the standard BKK model. Table 2 presents the different parameters used for these three models as well as for the alternative specifications discussed in the sensitivity analysis section.

The benchmark specification exhibits second moments that are different from the No Entry and No Extensive Margin models, but it does not deliver changes that solve any of the long-standing puzzles associated with international business cycles models subject to productivity shocks. Broadly speaking, a model with entry and fixed exporting costs shares most of the successes and failures of the standard BKK model in terms of the quantity and price anomalies.

Regarding aggregate volatilities, the benchmark model delivers statistics closer to the data and higher than the volatilities coming from the other two specifications that prevent entry. The volatility of output is 1.72, close to the value 1.67 reported in the data and higher than the value of 1.16 of the No Extensive Margin model. In addition, the trade balance, exports and all other domestic variables, with the exception of consumption, have higher volatilities and exhibit a better fit to the data than the alternative No Entry and No Extensive Margin models. These differences in volatility

[19]Data is from Heathcote and Perri (2002). The statistics related to the number of establishments and entry of new establishments is from own our calculations based on data from the Business Dynamics Statistics from the US Census Bureau. Data from the number of exporters is taken from Alessandria and Choi (2008). See details on the Appendix.

across specifications are mostly explained by the required adjustment in the parameterization of the stochastic process.

As we discuss in the next section, the parameterization of the exogenous process in the benchmark model exhibits a higher standard deviation on the innovations of the exogenous shock σ_v. This has a direct effect on the volatility of the output and all other variables. In addition, the model with entry delivers a higher volatility of labor because entry costs are denominated in labor units. Having an active extensive margin of production leads to a stronger effect of productivity shocks on hours worked. Once the local economy is hit by a positive shock, there is an increase in labor demand related to not only the intensive margin but also the expansion in the number of firms. Hence, there is a monotonous relationship between the volatility of entry and the volatility of hours worked arising from the assumption that the entry technology is labor intensive.

Regarding domestic variables, all specifications deliver the usual positive co-movement of consumption, labor and investment. Our benchmark model also delivers a positive correlation between the number of new entrants and output, mainly as a result of having an entry technology that is subject to productivity shocks. In the model, good times for scaling up production in existing firms coincide with good times for increasing the number of firms. This result is consistent with the positive correlation between cyclical changes in the number of establishments and output we compute from the data. Lee and Mukoyama (2007), Alessandria and Choi (2008) and Bilbiie, Ghironi and Melitz (2012) also document this feature.

Regarding trade variables, the four models capture the counter-cyclicality of the trade balance and reproduce the pro-cyclicality of exports but deliver a positive correlation between the number of exporters and output, which is at odds with the negative correlation at the product level documented by Alessandria and Choi (2008). Analogous to the case of entry, the positive correlation between the mass of exporters and output is driven by the assumption that fixed exporting cost are denominated in effective labor units.

The output-consumption anomaly is present across models: the cross-country correlation of output is lower than the cross-correlation of consumption. Although entry has a direct effect on the dynamics of GDP, it does not modify the underlying risk-sharing structure of the model, which is key for the smoother dynamics of consumption relative to output. Regarding production inputs, in the four specifications, labor is positively correlated across countries, while there is a negative relationship between investments in physical capital.

Additionally, all cases fail to reproduce the negative correlation between the relative consumption ratio and the real exchange rate, whether the latter is measured using the theoretical expression implied by the model or adjusted to resemble how price indices are constructed in practice by statistical offices. Having endogenous entry in the model does not help solve the Backus-Smith puzzle.

In summary, when comparing our benchmark model with other specifications, we find that having entry in the model, despite affecting the second moments of aggregate variables, does not ameliorate the main failures of the standard BKK model. We find, however, that the benchmark model exhibits a better fit regarding aggregate volatilities, mainly as a result of 1) a recalibration of the stochastic process that takes into account how the extensive margin of production affects the TFP of the model and 2) the effects on labor demand of having entry costs denominated in labor units.

When comparing the business cycle statistics for the No Entry and No Extensive Margin models, Table 3 shows that these two setups deliver almost indistinguishable second moments for aggregate variables with the exception of a few statistics pertaining to the number of exporters. The same is true when comparing the benchmark model with the No Export specification. This shows that fixed exporting costs alone have almost no effect on the overall dynamics of the model. This finding is in line with the findings of Alessandria and Choi (2007), which show that adding both fixed and sunk exporting costs to the standard model has no major effect on the dynamics and second moments of aggregate variables, with the exception of adding persistence to net exports.

5. Sensitivity Analysis

In this section, we investigate the interaction between various features of our calibration to assess the robustness of our quantitative results: 1) the adjustment of the marginal cost of entry, which disciplines the volatility of firm creation to replicate the volatility in the data; 2) the adjustment of the stochastic process for exogenous productivity to generate endogenous TFP dynamics with time series properties identical to those of the Solow residuals in the data; 3) the elasticity of substitution between domestic and foreign goods; 4) the denomination of entry and fixed exporting costs; 5) the assumption that entry costs are expensed and not capitalized into the economy's aggregate investment; and 6) the role of physical capital and endogenous labor. We conclude that none of these robustness exercises modify our main conclusion regarding the role of the extensive margin of production and exports for international business cycles, but we find them to be important regarding the quantitative performance of the model.

5.1. Specification of Entry Cost Function

To analyze the contribution of the elasticity of entry to the behavior of aggregate dynamics, we adopt an entry cost function that is linear in effective labor, as in Melitz (2003), Ghironi and Melitz (2005), and Bilbiie, Ghironi and Melitz (2012). In other words, we study our benchmark economy without imposing any convex adjustment penalty in the creation of new firms ($\gamma_e = 0$). We decompose the analysis into two scenarios, with and without adjusting the stochastic process for the exogenous shock. First, we re-calibrate the stochastic process as required by the linear specification of the entry cost function. This allows us to assess the full effect of the relaxation of entry cost. Secondly, we also consider a linear specification for entry costs but fix the parameters of the stochastic process at the values that we would have used if we had ignored the participation of endogenous variables in TFP. This step is helpful for isolating the effect of our assumption regarding the entry cost function.

The results for each case are reported in Tables 4 and 5, columns II and V, respectively.

Consider column V in table 5, which corresponds to the case in which the entry cost is linear and the productivity process is unadjusted. The specific values of the VAR that characterize the dynamics of the exogenous shock, ρ_z, σ_v and σ_{v*v}, are shown in Table 2. The upper block summarizes the coefficient of autocorrelation and the standard deviation and the correlation of the innovations for the model-based TFP. Consistent with the analysis of the previous section, the model tends to magnify the persistence of aggregate productivity as well as the correlation of the innovations and undermine its standard deviation. In other words, had the data been generated by a model with constant marginal costs of entry ($\gamma_e = 0$), we would have estimated a VAR for Solow residuals that is more persistent, more correlated across countries and less volatile.

In terms of second moments, column V in Table 5 shows that entry becomes significantly more volatile than in the benchmark and, therefore, than in the data, with its standard deviation reaching 7.19 times that of the GDP. However, the overall responsiveness of the economy tends to be hindered by the excess volatility in entry. The GDP, exports, investment and trade balance all become significantly less volatile than in the benchmark, while the volatility of labor increases, given that it is an input required for the expansion of the number of firms. Part of the reduction in volatility is due to our treatment of entry in the measurement of GDP. Recall that expenditures in entry are expensed and thus have no direct effect on the volatility of output. They do affect output in the next period, once the number of operating firms reflects the surge in entry, but this effect is downplayed by the high elasticity of substitution across varieties within a country[20].

A second source of the reduction in volatilities stems from the inverse relationship that arises between the volatility of investment and the volatility of entry. If entry costs are linear in labor, firm creation is stronger than in the benchmark case at the expense of the strength in the response of investment. The explanation for this behavior is twofold: 1) investment becomes a more costly mean to smooth the benefits of the shock over time and 2) a higher elasticity of entry implies that less labor is allocated to producing goods, which decreases the rate of return to physical capital. These

[20]Recall that with CES production function in the final good sector, the productivity gains implied by an increase in the number of varieties of producers is determined by the elasticity of substitution. The more substitutable goods are, the lower the benefit of increasing the number of firms. This can be seen from the the steady state expression for final output in equation (20).

effects are mitigated by the fact that the number of firms and the capital stock are complements in production. However, the mitigation is only partial and is outweighed by the first two forces.

As can also be observed from Column V, the specification of entry costs is less relevant for international correlations and the correlation between relative consumption and the exchange rate. Although the magnitudes differ to some extent, the second moments of the model with and without convex costs of entry display the same counterfactual properties: an excess correlation of consumption over output across countries and a positive correlation between real exchange rates and the ratio of consumption.

Column II in Table 4 reproduces the results for the linear entry cost case and adjusted shock process. The first three rows in the column contain the persistence, standard deviation and correlation values required by the calibration of the shock. In line with our previous findings, the resulting process needs to be far less persistent, significantly more volatile and virtually uncorrelated across countries to produce TFP dynamics in the model that are consistent with the Solow residuals in the data. This specification yields implausibly large magnitudes for the volatility of aggregate variables.

Regarding international correlations, the adjustment of the stochastic process has an important effect on the consumption and output correlations, making the latter larger than the former. This trend is consistent with the data, albeit too low when considered in absolute values. In addition, it mildly decreases the international correlation in firm entry and significantly decreases the international correlation in the number of exporters, both of which now become negative. These changes are explained by the large differences between the adjusted and unadjusted parameters of the shock process. This is true in particular for the persistence parameter, which falls to 0.42, and the standard deviation of innovation, which increases to 0.05.

Overall, this specification manages to solve the so-called output-consumption anomaly but does so at the expense of too much volatility in output, entry and all other macroeconomic variables.

5.2. Adjustment of the Shock Process

In the previous section, we discussed the importance of adjusting the shock process in the context of the linear entry cost model. For completeness, we repeat the analysis here for the case where the entry cost convexity parameter is calibrated to match the volatility of entry. We consider two scenarios for the stochastic process: the adjusted case, which is our benchmark calibration (column I, Table 4), and the unadjusted case, which follows the standard practice in the literature of imposing the estimates from the Solow residuals in the shock process directly (column IV, Table 5).

The first set of rows in column IV highlights the direction in which the model transforms the time series properties of the shock into TFP dynamics. The effect is identical to the results in the elastic entry margin case: the model magnifies the persistence of the shock process, undermines the volatility of its innovation and increases its correlation. This result can be understood using the same intuition gained by the decomposition of the endogenous components of TFP in Figure 1.

The comparison of absolute volatilities between columns I and IV also shows that the adjustment of the stochastic process is essential for the ability of the model to generate enough volatility in aggregate quantities. In the specification in which we do not adjust the shock, the output, exports and the trade balance have approximately half of the volatility of the benchmark model and of the data.

Unlike the case of linear entry costs, the adjustment of the stochastic process does not carry significant implications for international and domestic correlations because when the volatility of entry is calibrated, the difference between the parameter values of the adjusted and unadjusted shock is smaller, especially with respect to the persistence and the correlation of innovations.

5.3. Elasticity of Substitution and Trade Elasticity

We also verify whether our results are robust to our calibration regarding the elasticity of substitution among domestic varieties and across countries. These two parameters, especially the latter, which governs the Armington elasticity, are typically crucial for the dynamics of international business cycle models, as they determine the strength of the reallocation effect from the less productive to the more productive economy.

Table 4, column III, and Table 5, column VI, report the statistics for the adjusted and unadjusted specifications of the model under a calibration with higher mark-ups and higher elasticity between domestic and foreign goods. Our main finding regarding the role of the extensive margin of production and exports remains unchanged in this robustness exercise. None of the main failures of the model are reversed by the calibration that favors a lower θ and a higher ρ, although the dynamics and statistics of the model change. The higher Armington elasticity manifests in a stronger reallocation of factors across countries, such that domestic variables are more volatile and investment is more negatively correlated across countries. In the case of the adjusted TFP model, the higher ρ requires a lower persistence of the exogenous shock and higher volatility of the innovation parameter σ_v. The required TFP adjustment reinforces the higher volatility directly induced by the recalibration of the Armington elasticity and translates into a higher volatility of output, exports and trade balance relative to the benchmark model.

5.4. Effective Labor Units

Columns VII and VIII of Table 5, under the labels No Effective Entry and No Effective Export, explore the role of entry and fixed exporting costs being denominated in effective labor units. As mentioned earlier, this assumption implies that productivity shocks affect not only the production technology of intermediate goods but also the entry and exporting cost technology. We focus on the unadjusted TFP specification to isolate the direct effect of modifying this assumption without the required change in the calibration of the exogenous process.

When entry costs are not affected directly by productivity shocks, the role of the extensive margin of production is dampened. Entry is much less volatile and counter-cyclical, while the investment, exports and trade balance all display higher volatility. Labor responds in the same direction as entry, and its volatility relative to output falls to 0.24 from 0.61 in the benchmark case.

These statistics show that although departing from the assumption that entry is denominated in effective units of labor does not bring significant changes to the aggregate dynamics of the model or our main findings, it has an important effect on the relative weight of entry within the model and the statistics associated with it. In particular, it reverses the positive co-movement of entry and output of the benchmark model.

Column VIII shows that the assumption of effective labor units regarding fixed exporting cost plays no role in most of the dynamics of the model with the exception of the statistics related to the number of exporters. This specification, in which fixed exporting costs are denominated in non-effective labor units and hence productivity shocks do not directly affect the cost of becoming an exporter, delivers a slightly lower volatility of the number of exporters and makes this variable counter-cyclical. This last feature is consistent with the numbers reported by Alessandria and Choi (2008), although their statistic refers to cyclical variations of exported products rather than the number of exporters[21].

5.5. Capitalization of Entry Costs

Column IX reports the implied second moments of the model in the case in which entry costs are capitalized and accounted for as part of aggregate investment. In this case, we re-define investment as

$$I_t = K_{t+1} - (1-\delta)K_t + \tilde{v}_t N_{e,t}$$

This definition is consistent with the new revision of the NIPA accounts in which R&D expenditures will be recorded as investment. The last column reports the statistics of the model under this definition. We still report the volatility of the number of entrants, but under this specification, entry is reflected in the dynamics of aggregate investment. Note that in this case, there is no need to adjust the stochastic process. The endogenous TFP of the model exhibits the same stochastic properties of the exogenous process. We see this result as a validation of the exercise of adjusting the exogenous shock, as it is consistent with how entry costs have been historically treated in the national accounts framework.

[21]As show in this exercise, if we had chosen the specification of entry costs to match the negative correlation of number of exporters and output, all other second moments would have remain unchanged

In terms of the performance of the model, Column IX shows that this specification delivers a good fit of the model in terms of domestic volatilities. Unlike the benchmark model, it also predicts a positive co-movement of investment across countries due to the positive cross-country correlation of entry and its impact on investment dynamics. Regarding the consumption-output anomaly and the Backus Smith puzzle, this specification yields the same counterfactual result as the benchmark model.

5.6. The Role of Physical Capital and Endogenous Labor

Finally, we discuss the behavior of the economy under the assumption of exogenous labor supply and no physical capital stock in production, which is the setup of Ghironi and Melitz (2005). To compare our results with their findings, we report business cycle moments for the case in which the TFP process is unadjusted and entry and fixed exporting costs are linear.

The results show that abstracting from fluctuations in total hours worked and physical capital accumulation turns our economy into an unsatisfactory laboratory to study business cycles. This specification exhibits too little volatility. The volatility of the total output is less than a third of that in the data. Similarly, the volatility of exports and the trade balance is lower. The model also displays a counterfactual pro-cyclical trade balance absent in the benchmark specification. Using an endogenous labor supply and physical capital accumulation proves to be essential for not only the propagation and magnification of fluctuations in productivity but also the direction of capital flows and traded goods across countries.

Regarding the extensive merging of production, the volatility of the entry in this specification is approximately nine times the volatility of output, more than twice that in the data. Note that entry volatility is also too large in the full model with a linear cost function for entry (column V in Table 5). However, excluding capital and labor, entry volatility is even larger because firm creation absorbs the adjustment that would otherwise occur through investment in physical capital.

Lastly, the lack of endogenous labor and capital accumulation also manifests in the dynamics of the model-based TFP. The model magnifies the persistence of the shocks, implying an autocorrelation of the TFP of 0.99. It also reduces the estimated volatility of the process innovations. As in our previous discussion, the increase in persistence is explained by the dynamics of the number of firms, while the reduction in the volatility of innovations follows from the decline in the share of labor allocated to the production of goods.

6. Conclusion

Several studies have found that the extensive margin of production and exports plays a critical role in long-run changes to permanent shocks (i.e., trade liberalizations or financial market reforms). This paper studies the role of these margins in the propagation of high-frequency movements in aggregate productivity across countries.

Our conclusion is that the consideration of firm entry and exporting activity does not lead to significant deviations in the business cycle dynamics of the standard model for a representative firm. The puzzling counterfactual predictions of the benchmark BKK model, in terms of international co-movements and relative prices, carry over to the model with entry and fixed exporting costs.

In assessing the role of these margins, we highlight some of the challenges, as of yet unaddressed in the literature, that must be faced when working with business cycle models with an endogenous number of firms and propose how to address these challenges. One challenge stems from measuring GDP in accordance with NIPA practices, which used to treat sunk costs of entry and fixed costs of exporting as an expense rather than an investment. This measurement assumption implies that the dynamics of the number of firms and the fluctuations in labor accounted for by sunk and fixed costs are part of the economy's TFP. Under this view, a direct calibration of the stochastic process of the shock from Solow residuals is no longer legitimate. We overcome this problem by proposing a calibration such that the TFP implied by the model exhibits the same properties as the stochastic process estimated from the data.

We also show that a specification with linear entry costs, often used in the literature, over-predicts the volatility of entry and significantly reduces the fluctuations in other macroeconomic variables once they are measured consistently with how we map the model to the data. Adopting an ad-hoc but simple specification of entry costs, with a single parameter controlling the degree of convexity of the cost function, we are able to identify a parameter configuration that allows the model to replicate the volatility of entry. As a result, our conclusions are not driven by either over- or under-stating the cyclical fluctuations of the extensive margin of production.

A final caveat to the interpretation of our findings is in order. The claim that firm entry and endogenous exporter participation are not important for international business cycle applies only to the assumption that the origin of business cycle fluctuations lies in technology shocks. This result may not carry over to other types of shocks, especially if they generate a reallocation of production among firms. In this paper, we restrict our attention to productivity shocks in the context of a real business cycle model, leaving other sources of uncertainty for future research.

7. Acknowledgements

We are indebted with George Alessandria, Andy Atkeson, Christian Hellwig, Omar Licandro, Lee Ohanian, Vaidyanathan Venkateswaran, Mark Wright and participants at the LAMES/LACEA 2009 Conference, the St. Louis Fed seminar and the SED 2010 meetings in Montreal, for their helpful comments. We owe special thanks to Ariel Burstein for his guidance and suggestions and the useful comments of three anonymous referees.

8. References

[1] Alessandria, G., Choi, H., 2007. Do sunk costs of exporting matter for net export dynamics? The Quarterly Journal of Economics 122 (1), 289–336.

[2] Alessandria, G., Choi, H., 2008. The role of exporting and trade for entry over the business cycle. Unpublished manuscript Federal Reserve Bank of Philadelphia.

[3] Backus, D., Kehoe, P., Kydland, F., 1992. International real business cycles. Journal of Political Economy 100 (4), 745.

[4] Backus, D., Smith, G., 1993. Consumption and real exchange rates in dynamic economies with non-traded goods. Journal of International Economics 35 (3-4), 297–316.

[5] Baxter, M., Crucini, M., 1995. Business cycles and the asset structure of foreign trade. International Economic Review 36 (4), 821–854.

[6] Bernard, A., Eaton, J., Jensen, J., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93 (4), 1268–1290.

[7] Bernard, A., Jensen, J., 1999. Exceptional exporter performance: cause, effect, or both? Journal of International Economics 47 (1), 1–25.

[8] Bernard, A., Jensen, J., 2004. Why some firms export. Review of Economics and Statistics 86 (2), 561–569.

[9] Bilbiie, F., Ghironi, F., Melitz, M. J., 2012. Endogenous entry, product variety, and business cycles. Journal of Political Economy 120 (2), 304 – 345.

[10] Broda, C., Weinstein, D., 2006. Globalization and the gains from variety. Quarterly Journal of Economics 121 (2), 541–585.

[11] Burstein, A., Kurz, C., Tesar, L., 2008. Trade, production sharing, and the international transmission of business cycles. Journal of Monetary Economics 55 (4), 775–795.

[12] Corsetti, G., Dedola, L., Leduc, S., 2008. International risk sharing and the transmission of productivity shocks. Review of Economic Studies 75 (2), 443–473.

[13] Ghironi, F., Melitz, M., 2005. International trade and macroeconomic dynamics with heterogeneous firms. The Quarterly Journal of Economics 120, 865–915.

[14] Heathcote, J., Perri, F., 2002. Financial autarky and international business cycles. Journal of Monetary Economics 49 (3), 601–627.

[15] Heathcote, J., Perri, F., 2014. Handbook of international Economics. Vol. 4. Elsevier.

[16] Kehoe, P., Perri, F., 2002. International business cycles with endogenous incomplete markets. Econometrica, 907–928.

[17] Kollmann, R., 1996. Incomplete asset markets and the cross-country consumption correlation puzzle. Journal of Economic Dynamics and Control 20 (5), 945–961.

[18] Kydland, F., Prescott, E., 1982. Time to build and aggregate fluctuations. Econometrica 50 (6), 1345–1370.

[19] Lee, Y. T. M., 2008. Entry, exit and plant-level dynamics over the business cycle. Working Paper no.07-18, Fed. Reserve Bank Cleveland.

[20] Melitz, M., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71 (6), 1695–1725.

[21] Ravn, M., Uhlig, H., 2002. On adjusting the hodrick-prescott filter for the frequency of observations. Review of Economics and Statistics 84 (2), 371–376.

[22] Restuccia, D., Rogerson, R., 2008. Policy distortions and aggregate productivity with heterogeneous establishments. Review of Economic Dynamics 11 (4), 707–720.

[23] Schmitt-Grohé, S., Uribe, M., 2003. Closing small open economy models. Journal of International Economics 61 (1), 163–185.

9. Appendix

9.1. Data

We use the Business Dynamics Statistics (BDS) from the US Census Bureau to calculate our statistics on entry of new establishments. The BDS data is at annual frequency and starts in 1977. It covers all non-farm private establishments in all sectors of the economy and firms can be tracked by their size, age, and initial state of location in a particular state. The database contains information on entry, exit, job creation and job destruction at the establishment and firm level. For our numbers regarding volatility of entry we calculate the standard deviation of the cyclical deviations of new establishments using the HP filter. The smoothing parameter we use is 6.5 as suggested by Ravn and Uhlig (2002) for annual data. Given than in the model entrants become operational with one-period lag, when calculating the correlation between new establishments and output we lag the first variable in order to make our calculation consistent with the model. For the numbers of exporters we use as proxy the statistics reported by data Alessandria and Choi (2008) based on the figures by the Exporter Database for the number of exported products in the US. Although our reading of the model is in term of number of establishments Alessandria and Choi (2008) claim that the measure of the number of products is highly correlated to a measure based on the number of exporters from 1984 to 1992 from Bernard and Jansen (1999) and from the Exporter Database from 1996 to 2006.

9.2. Price Index and the Real Exchange Rate

The expression for the price index in the model is described by the following equation:

$$P_t = \left[N_{D,t}^{\frac{1-\rho}{1-\theta}} \tilde{p}_t^{1-\rho} + \left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}} \left(\tilde{p}_{x,t}^*\right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

As the economy experiences entry both to domestic and foreign markets, aggregate prices P_t could fluctuate just for this reason, even if average prices remained constant. GM and AC propose a price index where average prices are weighted by the total number of varieties. In particular, let $\tilde{P}_t = N_t^{\frac{1}{\rho-1}} P_t$, where $N_t = N_{D,t}^{\frac{1-\rho}{1-\theta}} + \left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}}$. Then,

$$\tilde{P}_t = \left[\frac{N_{D,t}^{\frac{1-\rho}{1-\theta}}}{N_t} \tilde{p}_t^{1-\rho} + \frac{\left(N_{x,t}^*\right)^{\frac{1-\rho}{1-\theta}}}{N_t} \left(\tilde{p}_{x,t}^*\right)^{1-\rho} \right]^{\frac{1}{1-\rho}} \tag{29}$$

This variety-adjusted price index is still at odds with the data as it implies that price indices update the share of domestic and exported goods every period, while in practice the consumption basket is revised by statistical agencies only sporadically. Furthermore, even for constant share of domestic and foreign goods, the average price of $\tilde{P}_t$ fluctuates in the model in response to changes in the productivity cut-off of exporters, leading to what could be associated with product substitution or quality adjustment in the data.

A more realistic definition regarding price indices is one where expenditure shares and the basket of foreign good are constant. Denoting such measure with $\hat{P}$, we have

$$\hat{P} = \left[\frac{N_{D}^{\frac{1-\rho}{1-\theta}}}{N} \tilde{p}_t^{1-\rho} + \frac{\left(N_{x}^*\right)^{\frac{1-\rho}{1-\theta}}}{N} \left(\tilde{p}_{x,t}^*\right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

where the share of expenditure in domestic and foreign goods, $\frac{N_D^{\frac{1-\rho}{1-\theta}}}{N}$ and $\frac{(N_x^*)^{\frac{1-\rho}{1-\theta}}}{N}$, is now time independent; and average foreign prices, $\tilde{p}_{x,t}^*$, are taken over a time independent set of varieties $z > z_x^*$. The measured real exchange rate consistent with this definition is given by:

$$RER = \frac{\left[\frac{(N_D^*)^{\frac{1-\rho}{1-\theta}}}{N^*}(\tilde{p}_t^*)^{1-\rho} + \frac{N_x^{\frac{1-\rho}{1-\theta}}}{N^*}\tilde{p}_{x,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\frac{N_D^{\frac{1-\rho}{1-\theta}}}{N}\tilde{p}_t^{1-\rho} + \frac{(N_x^*)^{\frac{1-\rho}{1-\theta}}}{N}\left(\tilde{p}_{x,t}^*\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}$$

Substituting average prices according to the optimal pricing rule of the firms, we can write RER as:

$$RER = \frac{\left[\frac{(N_D^*)^{\frac{1-\rho}{1-\theta}}}{N^*}(TOE_t)^{1-\rho} + \frac{N_x^{\frac{1-\rho}{1-\theta}}}{N^*}\left(\frac{\tau\tilde{z}_D}{\tilde{z}_x}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}{\left[\frac{N_D^{\frac{1-\rho}{1-\theta}}}{N} + \frac{(N_x^*)^{\frac{1-\rho}{1-\theta}}}{N}\left(TOE_t\frac{\tau\tilde{z}_D}{\tilde{z}_x^*}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}}$$

where TOE stands for terms of efficiency, and is defined as

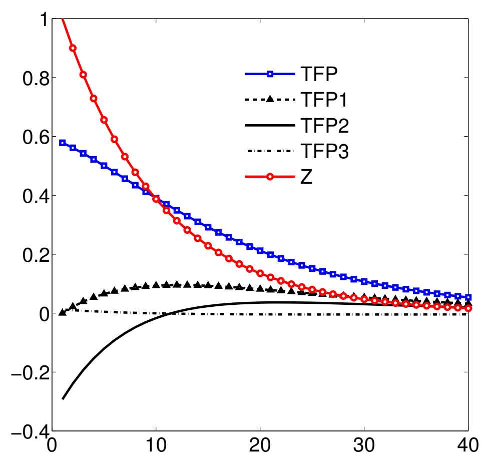
$$TOE = \left(\frac{r_t^*}{r_t}\right)^{\alpha}\left(\frac{w_t^*}{w_t}\right)^{1-\alpha}\frac{Z_t}{Z_t^*} \tag{30}$$

The expression captures the evolution of relative effective marginal costs of production in each country, and constitutes the sole source of fluctuation in the real exchange rate under the assumptions we made for the construction of the price indices. Having prevented any direct participation of entry and the exporting decision on the real exchange rate, these margins of adjustment will have implications for the dynamics of the real exchange rate only through their general equilibrium effect on wages and the rental rate of capital stock.

Table 1: Parameter Values Benchmark Model

Parameters	**Values**	**Targets**
Preferences	$\beta = 0.99$, $\gamma = 2$, $\mu = 0.38$	4% Interest Rates, 1/3 Hours Worked
Production	$\alpha = 0.36$, $\delta_K = 0.025$	Capital Income Share, Capital-Output Ratio
Distribution of Firms	$\kappa = 5.6$, $z_{min} = 1$, $\delta = 0.025$	Std. Dev. Log Sales, Establishment Exit Rate
CES Parameters	$\theta = 6$, $\rho = 1.5$	20% Mark-Up, Short-Run Trade Elasticity
Entry Costs	$f_e = 1$, $\gamma_e = 1.1$	Volatility of Entry
Exporting Costs	$\tau = 31$, $f_x = 0.0028$, $\gamma_x = 0$	Import Share 15%, $N_x/N_D = 21\%$
Adjustment Costs Bonds	$\eta = 0.0025$	Stationarity bond holdings
Stochastic Process	$\rho_z = 0.90$ $\sigma_v = 0.010$ $\sigma_{v^*v} = 0.105$	$\rho_{TFP} = 0.95$ $\sigma_\psi = 0.007$ $\sigma_{\psi^*\psi} = 0.29$

Figure 1: Decomposition of Measured TFP in the Model



Note: The line labeled TFP illustrates the overall dynamics of TFP in the model. The line labeled Z reproduces the exogenous component, which is the source of aggregate fluctuations in the economy. The model's TFP is broken down into three components that illustrate the contribution of changes of: the total number of firms (TFP1), the share of employment devoted to production (TFP2), and the average productivity of domestic firms (TFP3). All the lines are reported as percentage deviations from steady state.

Table 2: PARAMETER VALUES SENSITIVITY ANALYSIS

Adjusted TFP	
No Entry	$\gamma_e = 10,000$, $\rho_z = 0.97$, $\sigma_v = 0.007$, $\sigma_{v^*v} = 0.17$
No Export	$\gamma_x = 10,000$, $\rho_z = 0.90$, $\sigma_v = 0.012$, $\sigma_{v^*v} = 0.11$
No Extensive Margins	$\gamma_e = 10,000$, $\gamma_x = 10,000$, $\rho_z = 0.96$, $\sigma_v = 0.007$, $\sigma_{v^*v} = 0.18$
Linear Cost Entry (II)	$\gamma_e = 0$, $\rho_z = 0.42$, $\sigma_v = 0.050$, $\sigma_{v^*v} = 0$
High Elasticity (III)	$\theta = \rho = 3.8$, $\rho_z = 0.88$, $\sigma_v = 0.013$, $\sigma_{v^*v} = 0.24$
Unadjusted TFP	$\rho_z = 0.95$, $\sigma_v = 0.007$, $\sigma_{v^*v} = 0.29$
Benchmark (IV)	Baseline Parameters
Linear Cost Entry (V)	$\gamma_e = 0$
No Effective Entry (VI)	$\gamma_e = 0$, $\hat{f}_e w_t = \widetilde{v}_t$
No Effective Export (VII)	$d_{x,t} = Q_t/\theta \left[\rho_{x,t}(z)\right]^{1-\theta} \left(P_{X,t}/P_t^*\right)^{\theta-\rho} Y_t^* - w_t \hat{f}_x$
High Elasticity (VIII)	$\theta = \rho = 3.8$
Capitalized Entry (IX)	$I_t = K_{t+1} - (1-\delta)K_t + N_{e,t}\tilde{v}_t$, $GDP_t = Y_t + N_{e,t}\tilde{v}_t + TB_t$

Note: For the Adjusted TFP case we set the parameters of the process for exogenous productivity so that the model-based Solow residuals match the persistence and standard deviation of the Solow residuals in the data. The adjustment is done for each of the cases under consideration in the sensitivity analysis. For the Unadjusted TFP case we leave the parameters fixed at the estimated values from the data.

Table 3: BUSINESS CYCLES STATISTICS

	Data	Bench	No Entry	No Export	No Extensive Margins
		Volatility (stdv, %)			
GDP	1.67	1.72	1.18	1.72	1.16
Exports	3.94	1.54	1.04	1.52	1.01
TB/GDP	0.45	0.26	0.15	0.25	0.15
Import/Abs	0.55	0.05	0.04	0.04	0.04
		Volatility Relative to Output			
C	0.81	0.44	0.45	0.44	0.46
I	2.84	3.55	3.48	3.52	3.39
L	0.66	0.72	0.33	0.72	0.31
Nx	1.51	0.89	0.96	0.31	0.00
Ne	3.80	3.80	0.00	3.83	0.00
Q	2.32	0.34	0.35	0.34	0.35
		Correlation with Output			
C	0.86	0.99	0.98	0.99	0.98
I	0.95	0.98	0.98	0.98	0.98
L	0.87	0.96	0.99	0.96	0.99
Exports	0.32	0.49	0.59	0.49	0.60
TB	-0.49	-0.54	-0.51	-0.54	-0.51
Nx	-0.35	0.59	0.73	0.26	na
Ne	0.26	0.94	na	0.94	na
		International Correlations			
GDP	0.58	0.21	0.27	0.22	0.27
C	0.36	0.23	0.33	0.24	0.33
I	0.30	-0.08	-0.01	-0.08	-0.01
L	0.42	0.13	0.24	0.14	0.23
Nx	na	0.75	0.87	0.08	na
Ne	na	0.08	na	0.09	na
		Correlation Consumption Ratio (CH/CF)			
Q	na	0.95	0.98	0.95	0.98
TOE	-0.35	0.98	0.98	0.98	0.98

Note: The column "Bench" shows the business cycle statistics for our benchmark model and parameters. "No Entry" corresponds to the case where the total number of firms is held fixed, but in which the fraction of exporting firms changes over the cycle. "No Export" studies the case where there is entry to domestic markets, but the fraction of exporters is constant. The "No Extensive Margins" case assumes that both the total number of firms and the fraction of exporters are constant.

Table 4: SENSITIVITY ANALYSIS ADJUSTED TFP

	Bench (I)	Linear Cost (II)	High Elasticity (III)
ρ_Z	0.90	0.42	0.88
σ_v	0.010	0.050	0.013
σ_{v^*v}	0.11	0.00	0.24
	Volatility (stdv, %)		
GDP	1.72	4.58	1.91
Exports	1.54	4.87	2.39
TB/GDP	0.26	1.09	0.48
Imp/Abs	0.05	0.03	0.13
	Volatility Relative to Output		
C	0.44	0.40	0.45
I	3.55	4.02	4.15
L	0.72	1.42	0.75
Nx	0.89	1.05	1.23
Ne	3.80	9.79	2.86
Q	0.34	0.23	0.12
	Correlation with Output		
C	0.99	0.99	0.99
I	0.98	0.98	0.94
L	0.96	0.94	0.96
Exports	0.49	0.08	0.32
TB	-0.54	-0.64	-0.41
Nx	0.59	0.06	0.36
Ne	0.94	0.91	0.94
	International Correlations		
GDP	0.21	0.08	0.23
C	0.23	0.03	0.26
I	-0.08	-0.30	-0.19
L	0.13	0.00	0.22
Nx	0.75	-0.05	0.23
Ne	0.08	-0.03	0.19
	Correlation Consumption Ratio (CH/CF)		
Q	0.95	0.66	0.78
TOE	0.98	0.83	0.90

Note: "Linear Costs" corresponds to the case where the adjustment cost parameter for the entry cost function is set to zero, so that entry costs are linear in labor. "High Elasticity" shows results from increasing the elasticity of substitution between domestic and foreign goods from 1.5 to 3.8.

Table 5: SENSITIVITY ANALYSIS UNADJUSTED TFP

	Bench (IV)	Linear Cost (V)	High Elasticity (VI)	No Effective Entry (VII)	No Effective Export (VIII)	Cap. Entry (IX)	No Capital, No Labor (X)
ρ_{TFP}	0.98	0.99	0.99	0.94	0.98	0.95	0.99
σ_{ψ}	0.004	0.002	0.004	0.008	0.004	0.007	0.003
$\sigma_{\psi^*\psi}$	0.36	0.46	0.28	0.30	0.36	0.29	0.41
				Volatility (stdv, %)			
GDP	0.94	0.83	0.93	1.24	0.94	1.25	0.47
Exports	0.81	0.69	1.23	1.11	0.80	0.84	0.45
TB/GDP	0.10	0.08	0.27	0.16	0.11	0.10	0.03
Imp/Abs	0.03	0.03	0.10	0.04	0.05	0.03	0.17
				Volatility Relative to Output			
C	0.60	0.70	0.70	0.43	0.60	0.42	0.98
I	2.80	2.45	3.24	3.57	2.81	2.57	-
L	0.61	0.88	0.63	0.24	0.61	0.50	-
Nx	0.85	0.77	1.30	0.98	0.75	0.67	0.97
Ne	3.80	7.19	3.10	0.87	3.80	2.95	8.82
Q	0.36	0.38	0.13	0.33	0.36	0.27	0.59
				Correlation with Output			
C	0.99	0.98	0.99	0.96	0.99	0.98	1
I	0.98	0.97	0.92	0.97	0.98	0.99	-
L	0.96	0.87	0.95	0.99	0.96	0.98	-
Exports	0.65	0.70	0.22	0.58	0.65	0.60	0.97
TB	-0.51	-0.51	-0.43	-0.51	-0.52	-0.55	0.37
Nx	0.72	0.71	0.19	0.73	-0.45	0.69	0.98
Ne	0.93	0.79	0.92	-0.87	0.93	0.97	0.91
				International Correlations			
GDP	0.29	0.29	0.18	0.29	0.28	0.26	0.38
C	0.32	0.32	0.16	0.34	0.32	0.32	0.43
I	0.00	0.02	-0.38	0.01	-0.01	0.07	-
L	0.19	0.18	0.13	0.23	0.19	0.19	-
Nx	0.89	0.88	-0.08	0.87	-0.96	0.88	0.4
Ne	0.16	0.18	0.08	0.31	0.16	0.16	0.11
				Correlation Consumption Ratio (CH/CF)			
Q	0.96	0.92	0.59	0.99	0.96	0.96	0.97
TOE	0.99	0.98	0.77	0.98	0.99	0.99	1.00

Note: See the main text for a description of each specification.