# Master in Economics
# Lecture 3: International Risk Sharing

International Business Cycle

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# Risk Sharing and International Business Cycles

- Inspecting the data alone cannot answer what is the degree of risk sharing (efficiency) in international markets
- The Two-Country model can be used to assess whether the cross-country allocation of resources is efficient (Pareto sense)
- One key aspect in this discussion is the cross-country correlation of consumption.

# Cole-Obstfeld (1991)

- Ilustration of perfect risk sharing in a world **without** complete markets
- Two-country endowment economy in which country specializes in the production of one commodity
- Identical preferences over the two-commodities.

$$C^H = \left(c_1^H\right)^{\alpha} \left(c_2^H\right)^{1-\alpha}$$

- Demand functions:

$$c_1^H = \frac{\alpha Y}{p_1} \qquad c_2^H = \frac{(1-\alpha)Y}{p_2} \qquad c_1^F = \frac{\alpha Y^*}{p_1} \qquad c_2^F = \frac{(1-\alpha)Y^*}{p_2}$$

- Resource constraint: $c_1^H + c_1^F = Y \qquad c_2^H + c_2^F = Y^*$
- Prices: $p_1 = \frac{\alpha(Y+Y^*)}{Y} \qquad p_2 = \frac{(1-\alpha)(Y+Y^*)}{Y^*}$
- Allocation: $\frac{C^H}{C^{H+F}} = \frac{p_1 Y}{p_1 Y + p_2 Y^*} = \alpha \qquad \frac{C^H}{C^F} = \frac{\alpha}{1-\alpha}$

# International Risk Sharing (Quantities)

- Output, investment, and employment co-move positively and strongly across countries.
- The cross-country correlation of consumption is positive but smaller than the correlation of output.
- Net exports are not very volatile (their standard deviation is about one-third that of GDP) and are strongly countercyclical.

# Efficiency - Endowment Economy

- Discussion of efficiency in the BKK model: Heathcote and Perri (2014)
- Example Endowment economy with two-goods.

$$Max \left[ \kappa \frac{c_1^{1-\gamma}}{1-\gamma} + (1-\kappa) \frac{c_2^{1-\gamma}}{1-\gamma} \right] \tag{1}$$

subject to:

$$a_1 + a_2 = z_1$$

$$b_1 + b_2 = z_2$$

$$c_1 = \left[ \omega a_1^{\frac{\sigma-1}{\sigma}} + (1-\omega) b_1^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$c_2 = \left[ (1-\omega) a_2^{\frac{\sigma-1}{\sigma}} + \omega b_2^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

# Efficient Allocation

- FOC

$$\omega\kappa c_1^{-\gamma+\frac{1}{\sigma}} a_1^{-\frac{1}{\sigma}} = (1-\omega)(1-\kappa) c_2^{-\gamma+\frac{1}{\sigma}} a_2^{-\frac{1}{\sigma}}$$

$$(1-\omega)\kappa c_1^{-\gamma+\frac{1}{\sigma}} b_1^{-\frac{1}{\sigma}} = \omega(1-\kappa) c_2^{-\gamma+\frac{1}{\sigma}} b_2^{-\frac{1}{\sigma}}$$

- Log-linear system:

$$s\hat{a}_1 + (1-s)\hat{a}_2 = \hat{z}_1$$

$$(1-s)\hat{b}_1 + (s)\hat{b}_2 = \hat{z}_2$$

$$\left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_1 - \frac{1}{\sigma}\hat{a}_1 = \left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_2 - \frac{1}{\sigma}\hat{a}_2$$

$$\left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_1 - \frac{1}{\sigma}\hat{b}_1 = \left(-\gamma + \frac{1}{\sigma}\right)\hat{c}_1 - \frac{1}{\sigma}\hat{b}_2$$

# Efficient Allocation (Solution)

Solution

$$\frac{\hat{c}_1 - \hat{c}_2}{\hat{y}_1 - \hat{y}_2} = \frac{(2s-1)}{2\gamma(s-1) + 4s(\sigma\gamma - 1)(1-s) + 1}$$

The model fits the data

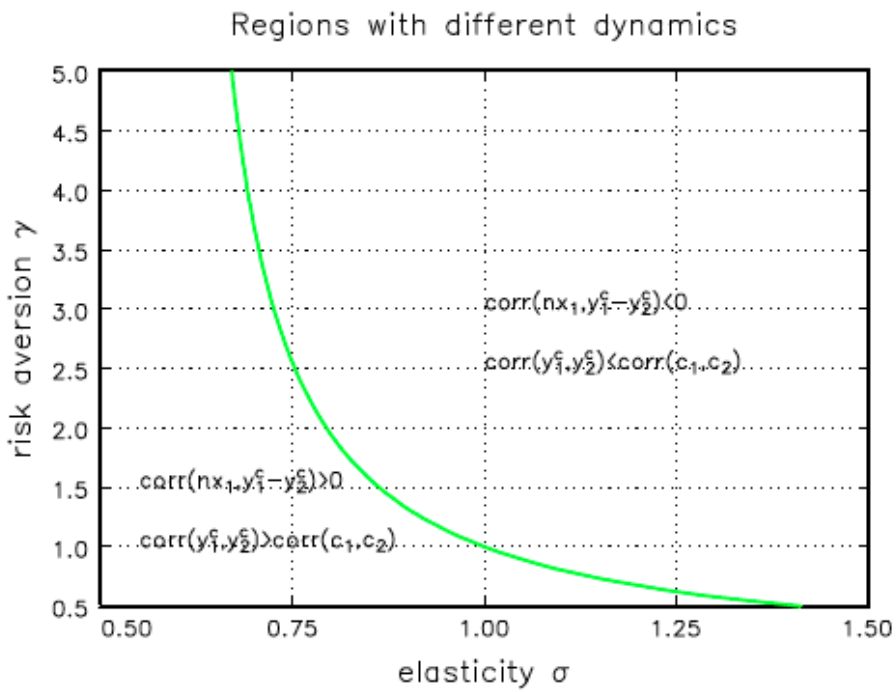
$$\frac{\hat{c}_1 - \hat{c}_2}{\hat{y}_1 - \hat{y}_2} > 1$$

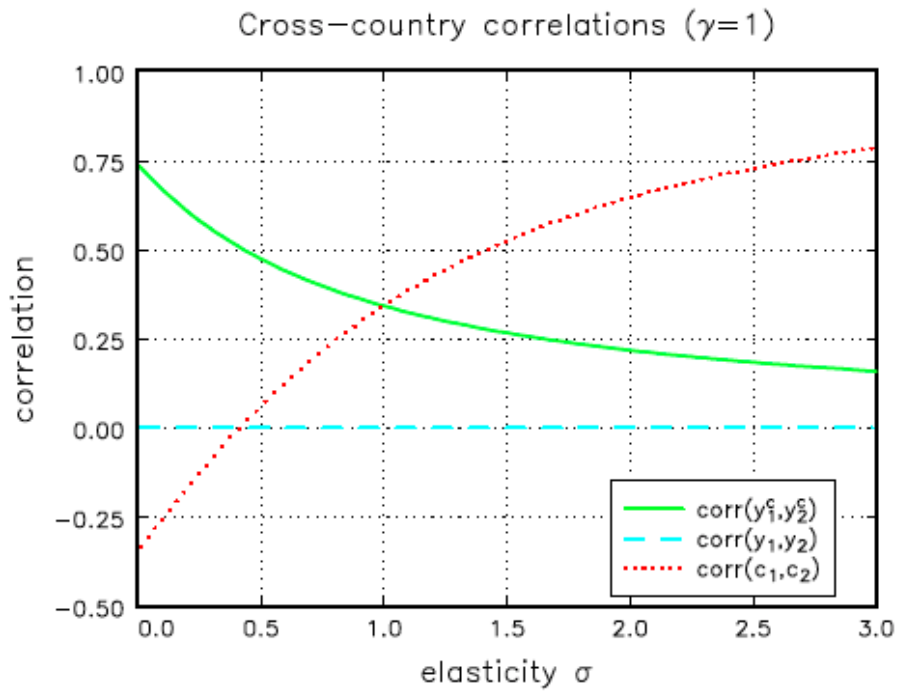
if

$$\sigma < \bar{\sigma} = \frac{1}{\gamma} - \left(\frac{1-\gamma}{2s\gamma}\right)$$

Note that when $s = \omega$, $\gamma = 1$, $\bar{\sigma} = 1$ (Cole-Obstfeld)

# Risk Aversion and the Elasticity of Substitution



# BKK with intermediate goods

$$Max\left[\sum_{i=1,2}\frac{\left(c_i^{\mu}(1-l_i)^{1-\mu}\right)^{1-\gamma}}{1-\gamma}\right] \tag{2}$$

subject to:

$$a_1 + a_2 = Z_1 F\left(k_1, l_1\right)$$

$$b_1 + b_2 = Z_2 F\left(k_2, l_2\right)$$

$$c_1 + k_1' - (1-\delta)\,k_1 = G_1 = \left[\omega a_1^{\frac{\sigma-1}{\sigma}} + (1-\omega)\,b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$c_2 + k_2' - (1-\delta)\,k_2 = G_2 = \left[\omega a_2^{\frac{\sigma-1}{\sigma}} + (1-\omega)\,b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$\begin{bmatrix} Z_{1t} \\ Z_{2t} \end{bmatrix} = \begin{bmatrix} \rho & \psi \\ \psi & \rho \end{bmatrix}\begin{bmatrix} Z_{1t-1} \\ Z_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{bmatrix}$$

# Efficient Conditions

$$U_{c_1} G_{a1} = U_{c_2} G_{a2}$$

$$U_{c_1} G_{b1} = U_{c_2} G_{b2}$$

$$\lambda_1 = \beta \lambda_1' \left[ G_{a_1} F_{k_1} + (1 - \delta) \right]$$

$$\lambda_2 = \beta \lambda_2' \left[ G_{b_2} F_{k_2} + (1 - \delta) \right]$$

$$U_{l_1} = U_{c1} G_{a_1} F_{l_1}$$

$$U_{l_2} = U_{c2} G_{b_2} F_{l_2}$$

# Efficiency in the BKK (revisited)

| | International correlations | | | | Domestic statistics | | |
|---|---|---|---|---|---|---|---|
| | $(y_1,y_2)$ | $(c_1,c_2)$ | $(x_1,x_2)$ | $(n_1,n_2)$ | % sd y | % sd $\frac{nx}{y}$ | corr($\frac{nx}{y}$,y) |
| 1. Data | 0.55 | 0.31 | 0.51 | 0.57 | 1.54 | 0.44 | -0.51 |
| Complete markets models | | | | | | | |
| 2. BKK (see Table 5) | 0.55 | 0.93 | -0.07 | -0.01 | 1.54 | 0.23 | -0.43 |
| 3. No spillovers: $\rho = 0.91, \psi = 0$ | 0.55 | 0.71 | 0.35 | 0.56 | 1.54 | 0.19 | -0.40 |
| 4. Separable utility: $\gamma = 1$ | 0.55 | 0.94 | 0.02 | 0.15 | 1.54 | 0.23 | -0.43 |
| 5. Low elasticity: $\sigma = 0.6$ | 0.55 | 0.88 | -0.08 | 0.10 | 1.54 | 0.28 | -0.47 |
| 6. All: $\rho = 0.91, \psi = 0, \gamma = 1, \sigma = 0.6$ | 0.55 | 0.35 | 0.39 | 0.71 | 1.54 | 0.47 | -0.46 |
| Bond economy model | | | | | | | |
| 7. BC: $\rho = 1, \psi = 0, \sigma = 5$ | 0.55 | 0.29 | -0.39 | 0.92 | 1.54 | 0.82 | -0.39 |

Notes: All data are from the OECD Quarterly National Accounts (GDP and components) and Main Economic Indicators (employment). The sample for the data statistics is 1960.1–2012.2. The variable y denotes real GDP, c denotes real consumption (both private and public), n denotes civilian employment, x denotes real gross fixed capital formation, nx/y denotes net exports over GDP (all nominal). All variables except net exports are in logs. All variables are HP filtered with a smoothing parameter of 1600. Statistics from the model are produced by simulating the model for the same numbers of periods as the the data and taking averages over 20 simulations. In lines 2 through 7 the standard deviation and correlation of shock innovations are calibrated to replicate the standard deviation of output and the international correlation of GDP.

# International Risk Sharing (Prices)

- The correlation with the consumption ratio $\left(\frac{c^H}{c^F}\right)$ and the real exchange rate ($Q = \frac{eP^*}{P}$) for various pairs of developed countries is close to zero, or even negative (**Backus-Smith Puzzle**)
- In the model the correlation is close to one.

$$\frac{U_{c_2}}{U_{c_1}} = \frac{G_{a1}}{G_{a2}} = Q$$

- With separable preferences:

$$\left(\frac{c_1}{c_2}\right)^{\gamma} = \frac{G_{a2}}{G_{a1}} = Q$$

# The Backus-Smith Puzzle

| | % sd e | % sd $\frac{c_1}{c_2}$ | corr($\frac{c_1}{c_2}$,e) |
|---|---|---|---|
| 1. Data | 6.39 | 0.97 | -0.21 |
| Baseline parameters: $\rho = 0.91, \psi = 0, \gamma = 1, \sigma = 0.6$ | | | |
| 2. Efficient allocations | 0.47 | 0.47 | 1 |
| 3. Bond Economy | 0.73 | 0.36 | 0.99 |
| 4. Financial Autarky | 3.15 | 0.02 | 0.79 |
| Very low elasticity: $\rho = 0.91, \psi = 0, \gamma = 1, \sigma = 0.38$ | | | |
| 5. Efficient allocations | 0.54 | 0.54 | 1 |
| 6. Bond Economy | 2.88 | 0.15 | -0.17 |
| High elasticity and pers. shocks: $\rho = 1, \psi = 0, \gamma = 1, \sigma = 5$ | | | |
| 7. Efficient allocations | 0.14 | 0.14 | 1 |
| 8. Bond Economy | 0.23 | 1.28 | -0.69 |

# The Backus-Smith Puzzle (II)

- Mussa (1986) documents that the real exchange rate is much more volatile between countries with flexible exchange rates
- Hess and Shin (2010) and Devereux and Hnatkovska (2011) report that the correlation between exchange rates and relative consumption is negative for countries (regions) with flexible exchange rates and positive for countries sharing a fixed exchange rate
- The connection between nominal exchange rates and fundamentals remains elusive

# Asset Pricing View Exchange Rates

- Using the standard model:

$$\frac{U_{c_2 t}}{U_{c_2 t}} = Q_t \qquad \frac{U_{c_2 t+1}}{U_{c_2 t+1}} = Q_{t+1}$$

- Brandt et al. (2006) propose the following decomposition:

$$q_{t+1} - q_t = log\left(\frac{U_{c_2 t+1}}{U_{c_2 t}}\right) - log\left(\frac{U_{c_1 t+1}}{U_{c_1 t}}\right) = log\left(m_{2t}\right) - log\left(m_{1t}\right)$$

- Colacito and Croce (2011) use Epstein-Zin preferences to distangle the SDF with movements in consumption.

# References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