Master in Economics
Lecture 4: IRBC and Heterogenous Firms

International Business Cycle

Jose Ignacio Lopez
HEC Paris

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Heterogeneity

- The extensive margin is at the core of trade models with love-for-variety
- Exporting firms tend to be larger and more productive
- Firm Heterogeneity helps to explain potential gains in productivity after trade liberalizations. Can they help to explain international business cycles?

Basic Facts about Exporting Firms

- Only 21% of manufacturing plants in the US export to other markets

- Exporting firms serve both domestic and external markets. Moreover, around 2/3 of exporters sell than 10% of their production in foreign markets

- Exporting firms are larger (measured in assets, labor, production) and more productive.

- These facts suggest that within industry reallocation can be an important force in trade liberalizations

Heterogenous Firms and the Aggregate Economy

- Hopenhayn (1992) studies a model with heterogenous firms, entry and exit and establishes the conditions for the existence of stationary equilibrium under perfect competition
- Melitz (2003)uses a heterogenous firms to account for the reallocation of factors within an industry after a trade liberalization

Melitz (2003) -Demand

- A CES utility function over a continuum of goods index by$z \in Z_t$

$$C_t = \left[\int_{z \in Z_t} c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

  Z_t is the set of available varieties and θ is the elasticity of substitution.

- Under perfect competion, the demand for an specif variety

$$c_t(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} C_t$$

- Aggregate Price:

$$P_t = \left[\int_{z \in Z_t} p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$$

Production

- Continuum of Firms. Labor is the only factor of production
- Firms have different levels of productivity: $c_t(z) = zl(z)$ and have a fixed production cost f.
- Wages are normalized to one. The total cost of the firm is: $TC(z) = f + c/z$
- Optimization problem $max \quad d(z) = p(z)\,c(z) - \frac{c(z)}{z} - f$. Using the demand function:

$$p(z) = \frac{\theta}{\theta - 1}\frac{1}{z}$$

- If $PC = R$, revenues of a specific firm: $r(z) = \left(\frac{\theta}{\theta-1}\right)^{1-\theta}(Pz)^{\theta-1}R$
- For any two firms, z_1 and z_2, $\frac{r(z_1)}{r(z_2)} = \left(\frac{z_1}{z_2}\right)^{\theta-1}$

Aggregation

- An equilibrium in this economy is fully characterized by a mass of firms (varieties) M and a distribution of productivities $\mu(z)$
- The aggregate price index is:

$$P_t = \frac{\theta}{\theta - 1} M^{\frac{1}{1-\theta}} \left[\int_0 z^{\theta-1} \mu(z)\, dz \right]^{\frac{1}{1-\theta}}$$

- We can define the average productivity of firms:

$$\tilde{z} = \left[\int_0 z^{\theta-1} \mu(z)\, dz \right]^{\frac{1}{1-\theta}}$$

- Use this as summary statistic:

$$P = \frac{\theta}{\theta - 1} M^{\frac{1}{1-\theta}} \tilde{z}$$

$$C = \frac{\theta - 1}{\theta} M^{\frac{\theta}{1-\theta}} \tilde{z}$$

Endogenous Distribution

- There is a large pool of prospective entrants
- Firms pay a sunk intial cost of entry f_e in order to draw from the common distribution $g(z)$ with positive support. $G(z)$ denotes the cummulative distribution
- Conditional on the draw firms decide whether to stay and produce or to leave the market.
- There is a constant probability of exogenous death δ
- The value of a firm is: $v(z) = \sum_{t=0} (1-\delta)^t d(z) - f_e$
- Zero-profit will determine a productivity threshold

$$\mu(z) = \left\{ \begin{array}{cc} \frac{g(z)}{1-G(\bar{z})} & if \quad z \geq \bar{z} \\ 0 & otherwise \end{array} \right\}$$

$$\tilde{z} = \left[\frac{1}{1-G(\bar{z})} \int_{\bar{z}} z^{\theta-1} g(z)\, dz \right]^{\frac{1}{1-\theta}}$$

Zero Profit Condition

- Zero profit implies: $r(\bar{z}) = \left(\frac{\theta}{\theta-1}\right)^{1-\theta} (P\bar{z})^{\theta-1} R = f$
- Ratio of revenues: $\frac{r(\tilde{z})}{r(\bar{z})} = \left(\frac{\tilde{z}}{\bar{z}}\right)^{\theta-1}$
- Profits average firm:

$$d(\tilde{z}) = r(\tilde{z}) - f = f\left(\left(\frac{\tilde{z}}{\bar{z}}\right)^{\theta-1} - 1\right)$$

- Ex-ante value of firm:

$$v_e = (1 - G(\bar{z})) \sum_{t=0} (1-\delta)^t d(z) - f_e = (1 - G(\bar{z})) \frac{d(\tilde{z})}{\delta} - f_e$$

- Free entry implies:

$$v_e = 0$$

$$d(\tilde{z}) = \frac{\delta f_e}{1 - G(\bar{z})}$$

Steady-State

- The mass of entrants M_e
- In steady-state:

$$[1 - G(\bar{z})] M_e = \delta M$$

- Labor-Market Clearing

$$L = L_p + L_e$$

$$L_e = M_e f_e = M_e \frac{[1 - G(\bar{z})] d(\tilde{z})}{\delta} = \delta M \frac{[1 - G(\bar{z})] d(\tilde{z})}{\delta} = \prod$$

International Trade

A CES utility function over a continuum of goods index by$z \in Z_t$

$$C_t = \left[\int_z a_t(z)^{\frac{\theta-1}{\theta}} dM_t(z) + \int_z x_t^*(z) b_t(z)^{\frac{\theta-1}{\theta}} dM_t^*(z) \right]^{\frac{\theta}{\theta-1}}$$

$$a_t(z) = \left[\frac{p_{a_t}(z)}{P_t} \right]^{-\theta} C_t \tag{1}$$

$$b_t(z) = \left[\frac{p_{b_t}(z)}{P_t} \right]^{-\theta} C_t \tag{2}$$

$$P_t = \left[\int_z p_{a_t}(z)^{1-\theta} dM_t(z) + \int_z x_t^*(z) p_{b_t}(z)^{1-\theta} dM_t^*(z) \right]^{\frac{1}{1-\theta}} \tag{3}$$

Production

- Continuum of Firms. Labor is the only factor of production
- Firms have different levels of productivity: $c_t(z) = zl\,(z)$ and have a fixed production cost f.

$$d(z) = \max_{p_{a_t}, p^*_{a_t}, a(z), a^*(z), x, l} p_a(z)a(z) + xp^*_a(z)a^*(z) - l - f` - xf_x$$

$$a(z) + x\tau a^*(z) = zl(z)$$

- Profit maximization problem of each variety producer gives:
- $p(z) = \frac{\theta}{\theta-1}\frac{1}{z} \quad p^*_a(z) = \frac{1}{\varepsilon}\frac{\theta}{\theta-1}\frac{\tau}{z} = \frac{1}{\varepsilon}\tau p(z)$
- $d(z) = \frac{1}{\theta}\left[\frac{p(z)}{P}\right]^{1-\theta} C_t - f \quad d_X(z) = Q_t\left[\frac{p^*_a(z)}{P}\right]^{1-\theta} C^* - f_x$

Entry into Domestic and External Markets

- Zero-profit will determine a productivity threshold

$$\mu(z) = \left\{ \begin{array}{ccc} \frac{g(z)}{1-G(\bar{z})} & if & z \geq \bar{z} \\ 0 & otherwise & \end{array} \right\}$$

$$\tilde{z} = \left[\frac{1}{1 - G(\bar{z})} \int_{\bar{z}} z^{\theta-1} g(z)\, dz \right]^{\frac{1}{1-\theta}}$$

- The exporting costs will determine a threshold for exporting

$$Q_t \left[\frac{p_a^*(z_x)}{P} \right]^{1-\theta} C^* = f_x$$

- The probability of exporting (conditional on succesful entry):

$$x = \frac{1 - G(\bar{z}_x)}{1 - G(\bar{z})}$$

Aggregation

- As before we can compute the average productivity of all firms and exporting firms
- The aggregate price index is:

$$P = \frac{\theta}{\theta - 1}\left[\tilde{z}_d^{1-\theta} + \tau^{1-\theta}\tilde{z}_x^{*1-\theta}\right]^{\frac{1}{1-\theta}}$$

- We can define the average productivity of firms:

$$\tilde{z} = \left[\int_{\bar{z}} {z_d}^{\theta-1} M_d \mu(z)\, dz\right]^{\frac{1}{1-\theta}}$$

$$\tilde{z}_x = \left[\int_{\bar{z}_x} {z_x}^{\theta-1} M_x \mu(z)\, dz\right]^{\frac{1}{1-\theta}}$$

$$M = M_d + xM_x$$

Impact of Trade

- What happens when the economy moves from the closed economy to the open economy?
- Firms find a new source of profits. Only more productive firms export to the foreign market. Higher profits induce higher entry (the value of the average firms increases)
- There is higher labor demand in the economy. As labor supply is fixed, wages increase forcing the least productive firms to exit.
- Aggregate prices fall (welfare improves) because of the positive reallocation torwards the most productive firms.

Ghironi and Melitz (2005)

- Ghironi and Melitz present a general equilibrium, two-country model with heteregeneous firms, which face sunk entry cost in the domestic market and both fixed and variables export costs (not fixed costs of production)
- This paper goes along in the tradition of trade literature related to the Harrod-Balassa-Samuelson effect
- The HBS effect can be defined as: the observation that consumer price levels in wealthier countries are systematically higher than in poorer ones
- The classical explanation for this phenomenon has been that productivity growth-rates vary more by country in the traded goods' sectors than in other sectors (the Balassa-Samuelson hypothesis).

GM Model

- In this model the division between traded and nontraded sectors is endogeneously determined and evolves over time
- Positive aggregate productivity shocks, expand the traded sector and translates into higher domestic prices. Therefore, the model replicates the HBS effect without relying on specific shocks to the traded sector
- The inclusion of per-unit export costs also allow the model to explain for persistent deviations from PPP, which also show up in cross-country price differences for tradable goods (Engel 1993, 1999).

Households

- Households solve (similarly for foreign household):

$$\max E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\gamma}}{1-\gamma} \right]$$

$$B_{t+1} + \tilde{v}_t N_{H,t} x_{t+1} + C_t = (1 + r_t) B_t + (\tilde{d}_t + \tilde{v}_t) N_{D,t} x_t + w_t L$$

$$\int_{\omega \in \Omega_t} p_t(\omega) c_t(\omega) dw = P_t C$$

$$C_t = \left[\int_{\omega \in \Omega_t} c_t(w)^{\frac{\theta-1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}$$

- $\tilde{v}_t$: date t price of claim to future profit stream of the mutual fund.
- $\tilde{d}_t$: average total profit (to be defined later)
- $N_{H,t} \equiv N_{D,t} + N_{E,t}$ Survivers: $N_{D,t+1} = (1 - \delta) N_{H,t}$.

Production Side

- Ex ante identical firms pay fixed entry cost of $f_{E,t}$ ($f^*_{E,t}$) *effective* labor units.→ $\frac{W_t f_{E,t}}{P_t Z_t}$ $\left(\frac{W^*_t f^*_{E,t}}{P^*_t Z^*_t}\right)$
- Upon entry, productivity is drawn from $G(z)$, $z \in [z_{\min}, \infty)$ (identical distribution for foreign firms).
- Relative productivity is kept until death, which occurs with probability δ
- Fixed per-period export cost of $f_{X,t}$ ($f^*_{X,t}$) units of effective labor→ $\frac{W_t f_{X,t}}{P_t Z_t}$ $\left(\frac{W^*_t f^*_{X,t}}{P^*_t Z^*_t}\right)$
- Iceberg Cost: $\tau_t \geq 1$ ($\tau^*_t \geq 1$)

Optimization Problem

- Profit maximization problem of each variety producer gives:
- $P_{D,t}(z) = \frac{\theta}{\theta-1}\frac{W_t}{Z_t z}$; $\quad P_{X,t}(z) = \frac{1}{\varepsilon_t}\frac{\theta}{\theta-1}\frac{\tau_t W_t}{Z_t z} = \frac{1}{\varepsilon_t}\tau_t P_{D,t}(z)$
- Expressing prices in real terms, relative to Price Index in destination market:

$$\rho_{D,t} = \frac{P_{D,t}(z)}{P_t} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}; \quad \rho_{X,t} = \frac{P_{X,t}(z)}{P_t^*} = Q_t^{-1}\tau_t \rho_{D,t}(z)$$

- where $Q_t = \frac{\varepsilon_t P_t^*}{P_t}$ denotes the real exchange rate.
- Similarly for the foreign country

Profits and Exporting Choice

- Profits in real terms relative to Price Index of where the firm is located (similarly for foreign firms):

$$d_{D,t}(z) = \frac{\Pi_D(z)}{P_t} = \rho_{D,t}(z)c_{D,t}(z) - \frac{w_t}{Z_t z}c_{D,t}(z)$$

$$d_{D,t}(z) = \frac{1}{\theta}\left[\rho_{D,t}(z)\right]^{1-\theta} C_t$$

$$d_{X,t}(z) = Q_t\rho_{X,t}(z)c^*_{X,t}(z) - \frac{w_t}{Z_t z}c^*_{X,t}(z) - \frac{w_t}{Z_t z}f_{X,t}$$

$$d_{X,t} = Q_t\left[\rho_{X,t}(z)\right]^{1-\theta} C^*_t - \frac{w_t}{Z_t z}f_{X,t}$$

  - Export Decision (similarly for foreign country):
  - A firm with productivity z exports $\iff z \geq z_{X,t}$, where $z_{X,t} = \inf\{z : d_{X,t}(z) > 0\}$
  - Endogenously determined non-traded sector: ex-ante each variety is tradeable, but some will not be traded ex-post

Firm Averages

- In every period, a mass $N_{D,t}$ ($N^*_{D,t}$) produces in each country
- $N_{X,t} = [1 - G(z_{X,t})]\, N_D\,, \quad N^*_{X,t} = \left[1 - G(z^*_{X,t})\right] N^*_D$
- $\tilde{z}_D = \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{X,t} = \left[\int_{z_{X,t}}^{\infty} \frac{1}{1-G(z_X)} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}$ (analogoulsy for the foreign country)
- Convenient definition of averages allows for:
- $\tilde{d}_{D,t} = d_{D,t}(\tilde{z}_D)$ where $\tilde{d}_{D,t} = \int_{z_{\min}}^{\infty} d_{D,t}(z) dG(z)$
- $\tilde{d}_{X,t} = d_{X,t}(\tilde{z}_{X,t})$ where $\tilde{d}_{X,t} = \int_{z_{x,t}}^{\infty} \frac{1}{1-G(z_{X,t})} d_{X,t}(z) dG(z)$
- Thus $\tilde{d}_t = \tilde{d}_{D,t} + [1 - G(z_X, t)]\, \tilde{d}_{X,t}$ represent average *total* profits of home producers (similarly for foreigners)

Aggregate Prices

- Recall definition of welfare based price index:

$$P = \left[\left[N_{D,t}\int_{z_{\min}}^{\infty} p_D^{1-\theta} + N^*_{X,t}\int_{z^*_{x,t}}^{\infty} \frac{1}{1-G(z^*_{X,t})} p^{*1-\theta}_{X,t}\right] dG(z)\right]^{\frac{1}{1-\theta}}$$

Definition of Averages is also such that;

$$\int_{z_{\min}}^{\infty} p_{D,t}(z)^{1-\theta} dG(z) = [p_{D,t}(\tilde{z}_D)]^{1-\theta}$$

$$\int_{z^*_{x,t}}^{\infty} \frac{1}{1-G(z^*_{X,t})} p^*_{X,t}(z)^{1-\theta} dG(z) = \left[p^*_{X,t}(\tilde{z}^*_{X,t})\right]^{1-\theta}$$

- Thus, Price Index becomes:

$$P_t = \left[N_{D,t}\left[p_{D,t}(\tilde{z}_D)\right]^{1-\theta} + N^*_{X,t}\left[p^*_{X,t}(\tilde{z}^*_{X,t})\right]^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

or

$$1 = \left[N_{D,t}\left[\rho_{D,t}(\tilde{z}_D)\right]^{1-\theta} + N^*_{X,t}\left[\rho^*_{X,t}(\tilde{z}^*_{X,t})\right]^{1-\theta}\right]$$

Free Entry

- Unbounded mass of prospective forward looking entrants in both countries
- Entrants at time t start producing at t+1
- Free Entry Condition: $\frac{w}{Z_t} f_{E,t} = \tilde{v}_t = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \tilde{d}_s$
- Law of Motion for mass of firms: $N_{D,t} = (1-\delta)\left[N_{D,t-1} + N_{E,t}\right]$

Parameterization

- Pareto Distribution: $G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^k$. Several simplifying implications
- k indexes dispersion of productivities: high values of k imply productivities are more concentrated towards $z_{\min}$ (draw picture)
- $\tilde{z}_D = v z_{\min}$, $\tilde{z}_{X,t} = v z_{X,t}$ and $\frac{N_{X,t}}{N_{D,t}} = 1 - G(z_{X,t}) = \left(\frac{v z_{\min}}{\tilde{z}_{X,t}}\right)^k$ where $v = \left\{\frac{k}{[k-(\theta-1)]}\right\}^{\frac{1}{\theta-1}}$
- Zero export profit condition for cutoff firm: $d_{X,t}(z_{X,t}) = 0$ implies
- $\tilde{d}_{X,t} = (\theta - 1)\left(\frac{v^{\theta-1}}{k}\right)\frac{f_{X,t} w_t}{Z_t}$

Real Exchange Rate

- Welfare-based price indexes don't correspond exactly to the CPI
- They can be break into $P_t = N_t^{\frac{1}{1-\theta}} \widetilde{P}_t$ where $N_t = N_{D,t} + N^*_{X,t}$.
- We can re-write a different price index as:

$$\widetilde{P}_t = \left[\frac{N_{D,t}}{N_{D,t} + N^*_{X,t}} \left[(\tilde{p}_D) \right]^{1-\theta} + \frac{N^*_{X,t}}{N_{D,t} + N^*_{X,t}} \left[\tilde{p^*_x} \right]^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Using this definition, it is possible to re-define the RER

$\widetilde{Q}_t = \frac{\varepsilon_t \widetilde{P}^*_t}{\widetilde{P}_t} \qquad Q_t = \widetilde{Q}_t \left(\frac{N_t}{N^*_t} \right)^{\frac{1}{1-\theta}}$

- It can be the case that if the product variety in the home market is large enough, we can have, for instance, that $Q_t > 1$ while $\widetilde{Q}_t < 1$

RER and TOT

- The terms of labor measures the relative cost of effective units of labor of one country in terms of the other: $TOL_t = \frac{\varepsilon(W_t^*/Z_t^*)}{(W_t/Z_t)}$
- A decrease in TOL, implies that labor in the home country has become relatively more expensive.
- The RER is affected by TOL as described by:

$$\widetilde{Q}_t^{1-\theta} = \frac{\left[\frac{N_{D,t}^*}{N_t^*}(TOL_t)^{1-\theta} + \frac{N_{X,t}}{N_t^*}(\tau_t \frac{\widetilde{z}_D}{\widetilde{z}_{X,T}})^{1-\theta}\right]}{\left[\frac{N_{D,t}}{N_t} + \frac{N_{X,t}^*}{N_t}(TOL_t \tau_t^* \frac{\widetilde{z}_D}{\widetilde{z^*}_{X,T}})^{1-\theta}\right]}$$

Changes in the RER

1. Changes in TOL translates into changes in home and domestic prices (potential source of differences for prices of nontraded goods across countries) ($\frac{P^*_{D,t}(z)}{P_{D,t}(z)} = \frac{\frac{\theta}{\theta-1}\frac{W^*_t}{Z^*_t z}}{\frac{\theta}{\theta-1}\frac{W_t}{Z_t z}}$)

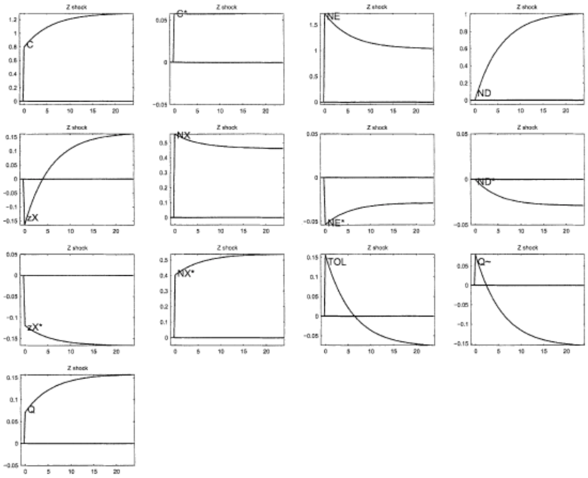
2. Changes in tradable prices (either by changes in tariffs or export productiviy cut-offs)

3. Expenditure switching between domestic and import varities

Calibration

Parameter	Value	Target/Source
β	0.99	standard RBC choice
θ	3.8	US plant and Macro Trade Data
γ	2	standard RBC choice
k	3.4	Std. Dev. log of US plant sales
δ	0.025	US job destruction rates
f_E	1	Only $\frac{f_X}{f_E}$ matters
f_X		21% of US plants export.

HBS Effect



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- We can re-write a different price index as:

$$\widetilde{P}_t = \left[\frac{N_{D,t}}{N_{D,t} + N^*_{X,t}} \left[p_{D,t}(\tilde{z}_D) \right]^{1-\theta} + \frac{N^*_{X,t}}{N_{D,t} + N^*_{X,t}} \left[p^*_{X,t}(\tilde{z}^*_{X,t}) \right]^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Using this definition, it is possible to definitionthe RER, closer to the CPI-measured one. $\widetilde{Q}_t = \frac{\varepsilon_t \widetilde{P}^*_t}{\widetilde{P}_t}$ $\quad Q_t = \widetilde{Q}_t \left(\frac{N_t}{N^*_t} \right)^{\frac{1}{1-\theta}}$
- It can be the case that if the product variety in the home market is large enough, we can have, for instance, that $Q_t > 1$ while $\widetilde{Q}_t < 1$

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