The importance of intangible capital for the transmission of financial shocks

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Abstract

This paper studies the role of intangible capital in the transmission of financial shocks in a general equilibrium model with two types of capital, tangible and intangible, and labor and financial frictions. We find that intangible capital, which cannot be used by financially constrained entrepreneurs as collateral, is key to generate labor market volatility in response to financial shocks. When hit by an adverse financial shock, entrepreneurs prioritize investment in pledgeable assets to offset the tightening of financial conditions. This results in a strong cutback in intangible investment, which in turn leads to a decline in the marginal product of labor, vacancies and employment. In an alternative specification -one without intangible capital- when hit by an adverse financial shock, entrepreneurs instead fund tangible investments by reducing their consumption. As a result, capital and the marginal product of labor fall less than in the model with intangible assets, resulting in a smaller decline of employment and output.

JEL classification: E24, E32, E44

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1. Introduction

Prior to the great recession, most of the literature focused primarily on the role of financial frictions in the propagation of productivity shocks. Since the crisis, there has been

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a lot of work devoted to study the effects of financial shocks, that is, shocks originating in the financial sector that directly affect the ability of firms to raise external funds. A common assumption in this literature has been that financially constrained firms use the total stock of productive capital as collateral. Although useful for highlighting the different mechanisms by which financial shocks affect the economy, this assumption is at odds with the fact that intangible capital, which is an increasingly important factor of production, cannot be used as collateral by virtue of its inalienable and illiquid nature.

This paper studies the role of intangible capital in the transmission of financial shocks to the real economy—in particular, to labor market outcomes, such as vacancies and employment—in the context of a dynamic general equilibrium model with two types of capital, tangible and intangible, and financial and labor market frictions. We find that intangible capital plays an important role in propagating financial shocks to labor markets by allowing entrepreneurs to smooth consumption at the expense of intangible investments. Facing an adverse shock, entrepreneurs cutback intangible investments, which in turn reduces the marginal product of labor and the incentive to hire workers.

Our economy is populated by relative impatient entrepreneurs and a representative household that supplies labor and provides funds to finance entrepreneurs’ productive activity. Entrepreneurs run a constant return to scale production function using labor and two types of capital—tangible and intangible. Because of the limited enforceability of contracts, entrepreneurs can borrow up to a fraction of the value of their stock of tangible assets.

An adverse shock to financing conditions, modeled as a tightening of the borrowing constraint, leads to a decline in production, as entrepreneurs have less access to external funds. This shock also tilts investment towards pledgeable capital, as entrepreneurs reduce intangible investment in favor of tangible assets, so as to mitigate the tightening of the borrowing constraint. The reduction in intangible capital leads to a decline in the marginal product of labor. Wages, which are set under the Nash bargaining mechanism, fall less than the marginal product of labor, reducing incentives to recruit. Entrepreneurs post fewer vacancies and the overall level of employment in the economy drops. The cutback in intangible investments and labor thus amplifies the initial decline in output triggered by the adverse shock to financing conditions.

Financial shocks allow the model to reproduce business cycle statistics and the observed dynamics of aggregate variables of the U.S. economy. These include labor market variables such as employment, vacancies and market tightness, and the procyclicality of the intangible-to-tangible capital ratio observed in the data. In line with previous work, the model fails to reproduce the volatility of labor market variables when subject to productivity shocks.

In order to highlight the role of intangible capital in the transmission mechanism of fi-
nancial shocks, we modify the model by keeping the ratio of intangible to tangible capital constant and equal to the steady-state value, so that entrepreneurs only optimize with respect to the pledgeable capital. This alternative version, which has the same steady-state as the benchmark model, shows that intangible capital is key to generating sufficient volatility in output, employment, vacancies and other labor market variables. In the alternative model, with only one endogenous capital (that is, tangible capital), an adverse financial shock has only a small effect on real activity.

Intangible capital amplifies the effect of financial shocks on the real economy inasmuch as it gives entrepreneurs additional flexibility to smooth consumption. In the model with only one type of capital, entrepreneurs prefer to cut consumption in order to sustain their investment in tangible capital and maintain similar levels of collateral, so as to not reduce their access to external funds. Given that in this specification productive capital does not fall much, the marginal product of labor remains relatively unchanged, instead of decreasing, as in the model with two types of capital. Without the drop in the marginal product of labor, the incentive to hire workers remains unaffected, and the one capital model exhibits little variation in employment and vacancies. In summary, a negative financial shock in a model with only tangible capital leads to a larger decrease in entrepreneurs’ consumption and half the decline in output, employment and vacancies, when compared to the model with intangible capital.

Finally, we discuss the role of search frictions in the labor market in the transmission of financial shocks in our model. We find that the interaction between intangible capital and labor market frictions— which introduce a wedge between wages and the marginal product of labor— is a key determinant of the amplification of financial shocks. When we modify the benchmark model by removing labor market frictions, a negative financial shock generates a large decline in output and investment, but not in labor. This is explained by the fact that, under this specification, the decline in the marginal product of labor caused by the fall in intangible investment is followed by an equally large reduction in wages, such that labor demand remains relatively constant.

The rest of the paper is organized as follows. In the next section, we discuss the contribution of the paper to the existing literature. Section (3) presents the model and equilibrium conditions. Section (4) presents the calibration and the aggregate dynamics of the model. In section (5), we discuss the role of intangible capital in financial shocks’ transmission mechanism. Finally, section (6) presents a sensitivity analysis of the key elements of the model in the transmission of shocks. The last section concludes.
2. Related Literature

This paper is related to three different strands in the literature. The first concerns research on the effects of financial shocks on the real economy. Examples of this strand are: Christiano, Motto, and Rostagno (2010), Jermann and Quadrini (2012), Kiyotaki and Moore (2012), Gilchrist and Zakrajsek (2012), Khan and Thomas (2013), Monacelli, Quadrini, and Trigari (2011), Petrosky-Nadeau (2014), Buera and Moll (2015), Perez-Orive (2016), Guerrieri and Lorenzoni (2017), Epstein, Shapiro, and Gómez (2017) and Wang (2017). This paper differs from this literature in that we discuss the role of intangible capital in the transmission of financial shocks to the real economy.

An important exception is the work by Perez-Orive (2016), which also includes intangible investment in a model with financial shocks. Our paper differs from his work in two important aspects: the presence of labor market frictions and the productive role of intangible capital. The model in Perez-Orive (2016) is silent regarding the effect of financial shocks on labor markets, inasmuch labor is exogenous in his framework. In addition, he assumes that intangible capital depreciates slowly and produces most of its output over the long-run. In line with this assumption, Perez-Orive (2016) finds a different role for intangible capital in the transmission of financial shocks than the one we discuss in this paper. An anticipated negative financial shock, generates in his model a short-run boom in the economy at the cost of lower output in the future, as firms are unwilling to sustain intangible investments, and increase their share of investment allocated to tangible capital. Conversely, in our model, a negative financial shock leads to a decline in output, driven by a cutback in intangible investment and a reduction in the marginal product of labor.

The second strand of the literature to which our work is relevant focuses on intangible capital. Corrado, Hulten, and Sichel (2009) and Corrado and Hulten (2010) document a secular increase in the aggregate investment of intangible capital in the U.S. since the 1950s. Hall (2001) and McGrattan and Prescott (2012) use the stock market and aggregate accounting data to infer from a standard investment model the dynamics of intangible capital in the U.S. Atkeson and Kehoe (2005) and Eisfeldt and Papanikolaou (2013) discuss the role of organization capital in the life-cycle investment and equity returns of firms. Falato, Kadyrzhanova, and Sim (2013) study external financing and cash holdings using a model featuring both financial frictions and investment in intangible assets, while Chen (2014) analyzes the effects of asset intangibility on firm dynamics when endogenous financial constraints are present. Finally, Gourio and Rudanko (2014) highlight the importance of introducing intangible capital to real business cycle models when intangibles capital is a factor of production. We contribute to this literature by studying the role of intangible capital in the transmission of financial shocks.

Finally, this paper relates to the literature examining cyclical changes in employment,
vacancies and market tightness using models with search frictions in labor markets. Since
the seminal work of Merz (1995), Andolfatto (1996) and the influential paper by Shimer
(2005), there has been a lot of work devoted to studying the quantitative performance
of the search and matching Mortensen and Pissarides (1994) model with productivity
shocks. Hornstein, Krusell, and Violante (2005) and Shimer (2010) present a summary
of this literature. We differ from this literature in that we analyze financial shocks in
addition to the standard productivity shocks, as in Petrosky-Nadeau (2014). We differ
from his work in that we extend the search and matching model to incorporate intangible
investment. Intangible investment helps to propagate financial shocks in labor markets
via changes in the marginal product of workers and the willingness of entrepreneurs to
hire workers. In Petrosky-Nadeau (2014) financial shocks have a direct effect on hiring
costs, as firms are assumed to require external funds to pay for job posting costs.

3. Model Description

We next outline our model, which is similar to one used in Shimer (2010), but includes
financial frictions and two types of capital: intangible and tangible.

3.1. Entrepreneurs

Entrepreneurs own capital and produce final goods that combined the former with la-
bor. We distinguish between two types of capital, tangible and intangible, which we
denote \( K_{T,t} \) and \( K_{I,t} \), respectively. This distinction becomes relevant when discussing the
borrowing problem of entrepreneurs, as only tangible capital can be used as collateral.
The number of employed workers involved in the production of the final good is denoted
by \( n_t \).

We implicitly assume that each entrepreneur runs his/her own firm, so we use these two
terms without distinction. The production technology exhibits constant returns to scale,
and is given by:

\[
F(K_{T,t}, K_{I,t}, n_t) = Y_t = Z_t K_{T,t}^{\alpha_{K_T}} K_{I,t}^{\alpha_{K_I}} n_t^{1-\alpha_{K_T} - \alpha_{K_I}}
\]  

(1)

where \( \alpha_{K_T} \) and \( \alpha_{K_I} \) are the income shares of tangible and intangible capital, respectively,
and \( Z_t \) is an aggregate productivity shock. Next-period’s employment, \( n_{t+1} \), is a function of
the surviving matches and new matches formed during the period. Entrepreneurs recruit
new workers by posting a total number of \( v_t \) vacancies each at a cost of \( q \) units of the final
good. Existing matches are destroyed at an exogenous separation probability denoted
by \( x \). We denote the matching probability from the firms’ point of view by \( \omega(\theta_t) \). This
matching probability is a function of labor market tightness \( \theta_t \), which is equal to the ratio
of vacancies versus unemployed workers: \( \theta_t = v_t / (1 - n_t) \).
Entrepreneurs maximize their expected discounted flow of instantaneous utility, derived from consumption, \( c^E_t \), by choosing their investment in tangible and intangible capital, the number of vacancies, and the amount of households’ external funds, \( B_{t+1} \), at an interest rate \( r_t \). Entrepreneurs are borrowing constrained and relatively impatient, as their discount factor, denoted by \( \gamma \), is smaller than the representative household’s discount factor, denoted by \( \beta \).

The optimization problem of an entrepreneur is thus summarized as:

\[
\max_{c^E_t, K_{T,t+1}, K_{I,t+1}, V_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \gamma^t \log \left( c^E_t \right) \tag{2}
\]

subject to the budget (3) and borrowing constrains (4), and the law of motion for employment (5):

\[
c^E_t + (1 + r_t)B_t + K_{T,t+1} - (1 - \delta_T)K_{T,t} + K_{I,t+1} - (1 - \delta_I)K_{I,t} + W_t n_t + q v_t = Y_t + B_{t+1} \tag{3}
\]

\[
B_{t+1}(1 + r_{t+1}) \leq \chi_t K_{T,t+1} \tag{4}
\]

\[
n_{t+1} = (1 - x) n_t + \omega(\theta_t) v_t \tag{5}
\]

where \( W_t \) is the wage earned by workers, and \( \delta_T \) and \( \delta_I \) are the depreciation rates of tangible and intangible capital, respectively. As entrepreneurs cannot commit to repaying their loans, they can only borrow up to the point where repayment equals a fraction \( \chi_t \) of the total value of their tangible capital. By its very nature, intangible capital cannot be seized in the event of a default, and thus cannot be used as collateral. The fraction of collateralized capital, \( \chi_t \), evolves stochastically, and reflects shocks to the terms of loans or, broadly speaking, the prevailing financial conditions of the economy. A negative shock to \( \chi_t \), for instance, implies that creditors are less willing to lend to entrepreneurs, relative to the same amount of collateral.

The timing of the constraint implies the following sequence of events: at the beginning of every period, entrepreneurs repay their debt, \( B_t (1 + r_t) \), and demand a new loan, \( B_{t+1} \), to finance production. They then produce, invest, pay job posting costs and workers, and consume. There is no default in equilibrium and entrepreneurs maximize their external financing, so the borrowing constraint always holds with equality.

Let \( \mu_t \) be the multiplier associated with the borrowing constraint (equation 4). The Euler equation for tangible investment is described by:

\[
1 - \mu_t \chi_t = \gamma E_t \left[ \frac{c^E_t}{c^E_{t+1}} \right] (r_{T,t+1} + (1 - \delta_T)) \tag{6}
\]
where $r_{T,t+1}$ is the return on tangible capital, in terms of the final good, and it is defined by the tangible capital-output ratio: $r_{T,t+1} = \alpha_{K_T} Y_{t+1}/K_{T,t+1}$. Note that the borrowing constraint’s multiplier directly affects tangible investment. In particular, whenever the constraint binds, entrepreneurs have an incentive to increase their tangible capital in order to relax their borrowing constraint.

The Euler equation for intangible investment is standard:

$$1 = \gamma E_t \left[ \left( \frac{c_t^E}{c_{t+1}^E} \right) (r_{I,t+1} + (1 - \delta_I)) \right]$$

(7)

where $r_{I,t+1}$ is the return on intangible capital, defined as: $r_{I,t+1} = \alpha_{K_I} Y_{t+1}/K_{I,t+1}$. The Euler equation for debt can be written as:

$$\mu_t = 1/(1 + r_{t+1}) - \gamma E_t \left( \frac{c_t^E / c_{t+1}^E}{\omega(\theta_t)} \right).$$

(8)

Given the differences in discount factors between the representative household and the entrepreneur, the multiplier $\mu_t$ is positive in steady-state, which implies that the borrowing constraint is satisfied with equality in the steady-state of the model.

Regarding recruiting decisions, the following Euler equation describes the trade-off faced by entrepreneurs:

$$q/\omega(\theta_t) = \gamma E_t \left[ \left( \frac{c_t^E / c_{t+1}^E}{\omega(\theta_t)} \right) (MPL_{t+1} + q(1 - x)/\omega(\theta_{t+1}) - W_{t+1}) \right]$$

(9)

where $MPL_t$ is the marginal product of labor in terms of the final good.

The left-hand side of the equation represents the cost of hiring one extra worker. Recall that the function $\omega(\theta_t)$ indicates the matching probability from the firms’ perspective, so its inverse, $1/\omega(\theta_t)$, represents the average number of vacancies required to hire one additional worker. Thus, the left-hand side of the Euler equation is equal to the cost of increasing the number of workers by one, in terms of output units. The right-hand side of the equation is the expected discounted return of increasing the size of the firm at the margin, and is equal to the marginal product of labor net of wage costs. The benefit of increasing the size of a firm includes, not only the additional output produced by the marginal worker hired in period $t$, but also the resources freed from recruiting at $t + 1$, while allowing the firm to maintain its original size at $t + 2$.

3.2. Households

The modeling of households follows Shimer (2010). There is a representative household composed of a unit measure of infinitely lived individuals with the same preferences for consumption and leisure (the disutility of working). In every period, each member of the household is either employed or unemployed. The utility flow a family member
obtains from nonworking activities while unemployed is represented by $\varphi$. In equilibrium a fraction of family members $n$ are employed, and a fraction $u = 1 - n$, are seeking a job, that is, they are unemployed.

The representative household supplies funds to entrepreneurs in the form of non-contingent one-period bonds, $D_t$. There is a household-level budget constraint, which states that the consumption of all family members, $c_t$, and the amount of funds supplied to firms should be equal to the total labor income of the household plus the return on the previous period’s bond holdings. The marginal utility of consumption is equalized across household members and under the assumption of separability between consumption and leisure, the household acts as if it has an instantaneous utility function equal to: $\log(c_t) - \varphi n_t$.

The optimization problem of the representative household can be written as:

$$\max_{c_t, D_{t+1}, E_0} \beta E_t \sum_{t=0}^{\infty} \beta^t \left[ \log(c_t) - \varphi n_t \right]$$ (10)

subject to the budget constraint:

$$c_t + D_{t+1} = n_t W_t + (1 + r_t) D_t$$ (11)

and the law of motion for employed workers given by:

$$n_{t+1} = (1 - x) n_t + f_t (\theta_t) (1 - n_t)$$ (12)

where $f_t$ is the probability that an unemployed worker finds a job, and is a function of the market tightness of the labor market, $\theta_t$. The law of motion for employed workers indicates that employment in the subsequent period is determined by the current period’s surviving matches plus the new matches formed in the labor market. The first-order optimal condition for the household problem is summarized by the standard Euler equation:

$$1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right]$$ (13)

3.3. Wage Determination

Following most of the literature, we use the Nash Bargaining solution to characterize a worker’s wage. As in most of the literature, we assume that firms do not internalize that investment and hiring decisions affect wages through changes in the marginal product of labor. Departing from this assumption requires solving the equilibrium wage using intra-firm bargaining, which affects the steady-state of the model, but not its business cycle.
properties. At the start of each period, each employed worker bargains with his or her employer over his or her wage. If bargaining fails, the match breaks up, the worker becomes unemployed, and the firm loses the output produced by the match. If bargaining succeeds, the worker is hired and is paid the equilibrium wage. Moreover, the worker and the firm anticipate agreeing on the equilibrium wage in any future period when they are matched.

Solving for the equilibrium wage gives us the following expression:

\[ W = \phi \left( MPL + q\theta \frac{\beta}{\gamma} + \frac{1 - x}{\omega(\theta)} q \left( 1 - \frac{\beta}{\gamma} \right) \right) + (1 - \phi) \phi c \]  

where \( \phi \in (0, 1) \) is the workers’ bargaining power. As in Shimer (2010), equilibrium wages are the weighted average of two terms: the marginal product of labor and the marginal rate of substitution between consumption and leisure. If a worker is employed, she or he can directly produce \( MPL \) units of output and, by marginally reducing the unemployment rate, frees up \( \theta \) units of the final good, since \( \theta \) is the aggregate vacancies-unemployment ratio. In our setup, firms are more impatient than households, which introduces two differences to the wage expression relative to the standard search and matching model. As mentioned earlier, the employment of an additional worker releases \( \theta \) units of the final good from recruiting, while keeping the recruiter-unemployment ratio constant. At the same time, increasing the size of the firm by one worker, frees up resources from recruiting by an amount equivalent to \( \frac{1 - x}{\omega(\theta)} q \). Inasmuch entrepreneurs and households discount the future at different rates, the present value associated with both of these savings is valued differently by the two types of agents. In the case of the vacancies-unemployment ratio, this determines that \( q\theta \) term is multiplied by the ratio of discount factors \( \frac{\beta}{\gamma} \). As for the firm’s savings, the difference in discount factors introduces a new term to the wage expression given by \( \frac{1 - x}{\omega(\theta)} q \left( 1 - \frac{\beta}{\gamma} \right) \). Note that if \( \beta \) and \( \gamma \) are equal, these additional terms disappear.

3.4. Equilibrium

An equilibrium in this economy is defined by a sequence of prices, \( \{ r_t, W_t \}_{t=0}^{\infty} \), and allocations to the household and entrepreneurs, such that these allocations are optimal and markets clear:

\[ f_t (\theta_t) (1 - n_t) = \omega (\theta_t) v_t \]  

We study the quantitative implications of intra-firm bargaining in an online appendix, following the work by Cahuc and Wasmer (2001). We find that after re-calibrating the steady-state of the model, this specification delivers business cycle statistics for aggregate variables almost identical to our benchmark. This result is in line with the findings by Krause and Lubik (2007).

\[ ^5 \text{See the Appendix for the full derivation} \]
\[ D_{t+1} = B_{t+1} \]  

(16)

We assume that the matching function exhibits constant returns to scale and that its elasticity with respect to market tightness is the same as workers’ bargaining power:

\[ m_t = \tilde{\omega} t_t^{\phi} v_t^{1-\phi} \]  

(17)

where \( m \) is the number of matches. Under this assumption the matching function from the point of view of firms can be expressed as: \( \omega (\theta) = \tilde{\omega} \theta^{-\phi} \).

3.5. Productivity and Financial Shocks

Productivity and financial shocks are modeled as autoregressive processes of first order:

\[ z_{t+1} = \rho_z z_t + \epsilon_{z,t+1} \]  

(18)

\[ \chi_{t+1} = \rho_\chi \chi_t + \epsilon_{\chi,t+1} \]  

(19)

where \( \rho_z \) and \( \rho_\chi \) govern the persistence of each shock and \( \epsilon_{z,t+1}, \epsilon_{\chi,t+1} \) are iid with standard deviations \( \sigma_z \) and \( \sigma_\chi \), respectively. These parameters cannot be set with steady state targets, so in the next section we discuss how we construct a time series for productivity and financial shocks from the data, following the procedure used by Jermann and Quadrini (2012). We transform the quarterly innovations into a monthly frequency in order to simulate the model. The details on how we obtain the monthly innovations from quarterly data can be found in the Appendix. Using the monthly innovations, we estimate equations (18) and (19).

4. Quantitative Analysis

Before discussing the role of intangible capital in the transmission of financial shocks, we discuss the quantitative properties of the baseline model. We calibrate and simulate the model at a monthly frequency, as do other papers that feature search frictions in the labor market. We use the policy functions to simulate the model, and then time-aggregate the variables to a quarterly frequency.\(^6\) We test the ability of the model to reproduce key features of the U.S. economy for the period 1980.Q1-2016.Q4 along two dimensions: standard real business cycle statistics and the dynamics of the model’s endogenous variables induced by the time series of productivity and financial shocks estimated from the data.

\[^6\]The model is solved by computing a log-linear approximation around the deterministic steady-state. Dynare codes available upon request.
4.1. Calibration

4.1.1. Preferences and frictions

Table 1 presents the list of parameter values used in our numerical simulation. The discount factor of the representative household, $\beta$, is set at 0.997, so as to match a real annual interest rate of 4%. We follow the literature, in particular the parameterization of Shimer (2010), regarding the key parameters for search frictions in labor markets. We set workers’ bargaining power, $\phi$, as equal to 0.5, and the exogenous separation rate, $x$, as equal to 0.034. The disutility parameter $\varphi$ and the parameter governing the matching efficiency, $\bar{\omega}$, are set to match the long-run average unemployment rate of 5%. We calibrate the cost of posting vacancies, $q$, so that recruiting costs are equal to four percent of one worker’s quarterly wage, in line with estimates by Silva and Toledo (2009).

The steady-state value of the share of collateralized capital, its unconditional mean, $\chi$, is set to match a steady state ratio of debt to quarterly GDP of 3.4 (in quarterly terms). This is the average ratio over the period 1980.Q1-2016.Q4 for the non-financial business sector based on data from the Flow of Funds for debt, and National Income and Product Accounts (NIPA), for GDP. The entrepreneur’s discount factor, $\gamma$, is set low enough to guarantee that the borrowing constraint is always binding and so as to match a rate of return of 24% of entrepreneurial investment, as documented by Moskowitz and Vissing-Jørgensen (2002).

4.1.2. Tangible and intangible capital

We calibrate the parameters related to the production technology in order to match the investment-to-output ratios of tangible and intangibles assets derived from NIPA. Since the 2013 revision, the Bureau of Economic Analysis (BEA) expanded the definition of the asset boundary to include innovative activities that correspond to intangible assets. This category, which is labeled “intellectual property products” and “intangible fixed assets” in the international-style NIPA aggregates, includes investments in research and development (R&D), software, entertainment, and literary and artistic originals, all of which are intangible by nature. With this new definition, NIPA provides estimates of investments in intangible assets from 1947 onward at both quarterly and annual frequency.

From the NIPA accounts we cannot infer the total income accrued from this type of intangible investment, so we set the intangible capital income share, $\alpha_{K_I}$, equal to 0.15, following estimates by Corrado, Hulten, and Sichel (2009) and as in Perez-Orive (2016). We set the depreciation of intangible capital, $\delta_I$, so as to target the 5% intangible investment to the output ratio for the sample period according to NIPA. We define tangible investment as the sum of other non-residential investments, including investments in structures, equipment, and changes in private inventories. The tangible capital income share, $\alpha_{K_T}$, is chosen in order to have a labor income share, $\alpha_n = 1 - \alpha_{K_T} - \alpha_{K_I}$, of 60% and we set
the depreciation rate for this type of capital, $\delta_T$, so as to match a ratio of 12% of tangible investment to output.

4.1.3. Stochastic processes

In order to calibrate the parameters governing the dynamics of the shocks presented in equations (18) and (19), we follow the procedure used by Jermann and Quadrini (2012). For productivity shocks, we use the standard Solow residual approach to recover $z_t$ from the data. Based on the production function (equation 1), we recover the time series for total factor productivity as:

$$\hat{z}_t = \hat{y}_t - \alpha_K\hat{k}_T - \alpha_L\hat{k}_I - (1 - \alpha_K - \alpha_L)\hat{n}_t$$

(20)

where all the variables (with hats) are in log-deviations from a linear trend. We use the perpetual inventory method to construct the series for tangible and intangible capital, and iterate forward using the data for intangible and tangible investment (Initial capital is computed as the investment in 1952.Q1 divided by the depreciation rate. However, given that our period of study goes from 1980.Q1 to 2016.Q4, the initial values do not affect our results). Data for output and tangible and intangible investment is from NIPA. As explained before, tangible investment includes structures, equipment and changes in private inventories, while intangible investment is the sum of R&D, software, entertainment, literary and investment on artistic originals. The labor input is measured using the number of civilian employment, 16 years and over, from the Bureau of Labor Statistics (BLS). Given the values for the $\alpha_K$ and $\alpha_L$ parameters, and the empirical series for $\hat{y}_t$, $\hat{k}_T$, $\hat{k}_I$, and $\hat{n}_t$, we can compute the $\hat{z}_t$ series.

To construct the series for the financial shock, we assume that the borrowing constraint (equation 4) holds, so we write the fraction $\chi$ as:

$$\chi_t = \frac{B_{t+1}(1 + r_{t+1})/Y_t}{K_{T,t+1}/Y_t}$$

(21)

Each term in this equation has a clear counterpart in the data. The numerator is the leverage of the private sector, which we compute using the Flow of Funds data. The denominator is the tangible capital-to-output ratio, which we construct using data from NIPA. We are thus able to compute the series for the financial shock and find its cyclical component, $\hat{\chi}_t$, by taking log deviations from a linear trend.

As in Jermann and Quadrini (2012), the validity of this procedure depends on the validity of the assumption that the borrowing constraint is always binding. We check this condition ex-post by feeding the constructed series into the model and checking whether the constraint is always binding.
Finally, we estimate the auto-regressive equations (18) and (19), after checking that the parameter estimates are significant and that the first-order lag is optimal according to the Akaike information criterion. We also estimate a vector autoregressive system in which both shocks are jointly estimated allowing for spill-over terms between the two shocks:

\[
\begin{pmatrix}
\hat{z}_{t+1} \\
\hat{x}_{t+1}
\end{pmatrix} = \begin{pmatrix}
\rho_{zz} & \rho_{zx} \\
\rho_{xz} & \rho_{xx}
\end{pmatrix} \begin{pmatrix}
\hat{z}_t \\
\hat{x}_t
\end{pmatrix} + \begin{pmatrix}
\epsilon_{z,t+1} \\
\epsilon_{x,t+1}
\end{pmatrix}
\]  

(22)

The results from this exercise are presented in table (2).

4.2. Business cycle dynamics

Table (3) presents the key statistics of the model and compares them with the data. We report statistics for the model when it is hit by either TFP or financial shocks, one at a time and using the estimates from equation (18) and (19). The volatility of the data is computed as the standard deviation of the cyclical component of the linear detrended quarterly series. To compute the volatilities implied by the model, we simulate the model at a monthly frequency and then time-aggregate the variables to a quarterly frequency.

The model fails to reproduce the volatility of employment variables when subject to only TFP shocks. The small employment volatility of the model with TFP shocks is not surprising in light of two results. First, as discussed by Shimer (2005, 2010), models with search frictions in the labor market and flexible wages (Nash bargain) fail to account for the volatility of labor markets variables when subject to TFP shocks, because changes in productivity affect both the relative bargaining position of workers and the marginal product of labor in a similar way, leaving the surplus of a match unchanged. Second, as shown by Olivella and Roldan (2011), in models with reproducible capital, general equilibrium effects dampen the response of real variables to productivity shocks, even in the presence of financial frictions.

Regarding the relative use of intangible and tangible capital, TFP shocks deliver a volatility to this ratio that is close to zero. Changes in TFP affect all production inputs uniformly, and thus generate little reallocation between the two types of capital.

Financial shocks allow the model to produce statistics different from those produced by TFP shocks, and in line with the data. The volatility of employment relative to output produced by financial shocks is three times greater than in the specification with TFP shocks, and much closer to the data (0.56 in the data versus 0.55 in the model). In addition, the specification with financial shocks helps to improve the correlation of labor market variables in line with the data, in particular, by reducing the correlation of these variables with labor productivity. The correlation between market tightness and labor productivity is 0.68 with financial shocks, which is closer to the 0.61 in the data than the 0.97 in the
case of productivity shocks. Something similar occurs with the correlation between the number of vacancies and labor productivity (0.64 in the data, 0.9 in the case of TFP shocks and 0.41 with financial shocks). Financial shocks reproduce a volatile and pro-cyclical intangible to tangible capital ratio, because when financial conditions tighten, changes in $\chi$ induce entrepreneurs to tilt investment in favor of collateral assets. The model with financial shocks produces a ratio of intangible-to-tangible capital with a higher volatility than that in the data (2.24 versus 1.26), which is not surprising given that the reallocation between intangible and tangible capital is key to the model’s propagation mechanism.

4.3. Dynamics induced by shocks

We also test the model’s fit by discussing its dynamics when fed with the observed realizations of productivity and financial shocks estimated for the U.S. economy for the period 1980.Q1-2016.IQ4. With this goal in mind, we conduct the following simulation. Starting with initial values of $\hat{z}_{1980.Q1}$ and $\hat{\chi}_{1980.Q1}$, we compute the quarterly innovations for our sample period going up to 2016.Q4. These are shown in Figure (1). The quarterly innovations are transformed into a monthly frequency, and then fed into the model to compute dynamics for output, employment, vacancies, and other key variables in the model. We then time-aggregate the monthly variables into quarters, so that they can be compared with the data. We do this for three different specifications of the model: one with only productivity shocks ($\hat{\chi}$ is kept constant at its unconditional mean value $\bar{\chi}$), a second with only financial shocks, and a third with both shocks, using our estimates of the autoregressive system in equation (22). Note that, although we use the actual sequence of shocks, the model’s agents do not perfectly anticipate them, as they forecast their future values every period using the autoregressive system described in equation (22). Finally, we verify that the multiplier of the borrowing constraint remains positive during the entirety of the simulation period, implying that the constraint is always binding.

Figure (2) plots the response of selected aggregate variables under the three different specifications: productivity shocks (dashed lines), financial shocks (dotted line), and both shocks (solid lines with circles), together with the corresponding data (solid black lines). Regarding output, TFP shocks capture most of its dynamics. Financial shocks do not generate enough output fluctuations during the first half of the sample, as a result of smaller innovations during the 1980s, but do appear to perform better during the second half. The specification with both shocks tracks output fluctuations quite well, though slightly overestimates their volatility at the end of the sample period.

With respect to labor market variables, TFP shocks alone do not capture any of the movements observed in the data, as they fail to generate enough volatility. Moreover, the drop in employment captured by TFP shocks during the last recession is barely noticeable relative to the fall observed in the data. On the contrary, the specification with financial
shocks accounts for a large fraction of the fluctuations in labor market outcomes during the sample. In particular, financial shocks generate sharp drops in vacancies, market tightness and employment in three of the four recessions: 1990-1991, 2001 and 2008-2009. Financial shocks also deliver realistic dynamics of debt-to-GDP, which cannot be replicated by productivity shocks. The model with financial shocks reproduces the dynamics of aggregate capital, as well as the investment composition of intangibles versus tangibles, although it does exhibit a greater volatility relative to the data for the latter.

4.4. Impulse-response functions

Before discussing the role of intangible capital in the model’s propagation mechanism, we discuss the impulse-response functions in order to analyze the mechanisms behind the propagation of financial shocks.

Figure (3) shows the impulse response functions to a negative one-standard deviation of a TFP shock (dotted line) versus that of a financial shock (solid line). As we use the impulse-response function mainly to understand the model’s propagation mechanism and to contrast the effect of financial shocks versus productivity shocks on the key variables of the model, we present them at a monthly frequency.

As shown by the impulse-response functions, TFP shocks impact output directly, and thus generate a large and persistent fall in output. However, the response of other variables such as employment, vacancies, aggregate capital and debt, is relatively weak.

Financial shocks have a very different impact on all variables. They produce a larger drop in employment and in the stock of debt and aggregate capital. A tighter borrowing constraint forces entrepreneurs to reduce their debt and scale down their operation. It also gives them incentives to reallocate resources towards the pledgeable asset $K_T$ at the expense of investment in the non-collateralized capital $K_I$, so as to offset the tighter borrowing constraint. The reallocation of investment towards tangibles, at the expense of intangibles, makes the ratio of intangible to tangible capital to drop after the shock, so it becomes pro-cyclical.

The decrease in $K_I$ is such that aggregate capital falls, reducing the marginal product of labor and thus the incentive to hire workers. Wages fall less than the marginal product of labor, reducing the incentive to hire workers. This leads to a fall in vacancies, the market tightness of the labor market, and, ultimately employment. Note that the smaller decline in wages relative to the marginal product of labor would not be present if labor markets were frictionless, as without search frictions, these two variables would move in tandem.

5. The role of intangible capital in the transmission of financial shocks

In this section, we analyze the role of intangible capital in the propagation of financial shocks. For this purpose, we compare our benchmark model with a specification where
the ratio of intangible to tangible capital is set exogenously constant at the steady-state level, such that the two models have the same steady-state and only differ in that in the alternative specification entrepreneurs have one less endogenous variable. Note that this exercise maintains the labor income share unchanged and matches the same steady-state targets of the baseline calibration.

With this change in the model specification, the Euler equation for intangible capital (equation 7) is not part of the equilibrium conditions. Consequently, entrepreneurs only make decisions regarding investment in tangible capital.

We compare the two models by looking at the impulse-response of selected endogenous variables to a one standard negative shock to financing conditions at a monthly frequency. We label the model without endogenous intangible capital as the one capital model (1K), and present the results in Figure (4).

The 1K model exhibits less amplification of the financial shock in terms of output, employment and other labor market variables. Unemployment, for instance, increases half as much in the 1K model as in the baseline model following the adverse shock to financing conditions. Relative to the benchmark model, the volatility of output, employment and vacancies in the 1K model is 0.21%, 0.17% and 0.40%, respectively. The marginal product of labor, which in the benchmark model drops a few periods after the shock, stays almost constant in the model featuring only one type of capital. The only two variables that exhibit higher volatility in the one capital model than in the benchmark one are consumption and the debt of entrepreneurs.

Why does adding intangible capital amplify the effect of financial shocks on output, investment and labor market variables? Should not this extra margin given by intangible capital help firms “smooth out” the shock?

In the benchmark model, entrepreneurs use the additional flexibility of having intangible capital to finance investment in pledgeable assets in order to offset the tightening of the borrowing constraint. They do this by cutting intangible investments without reducing consumption to the same extent as in the model with only one type of capital. The reduction of intangible investment, which causes the ratio of intangible to tangible capital to fall, is costly in terms of output and reduces the marginal product of labor, as labor and intangible capital are complimentary. The fall in the marginal product of labor reduces the incentive to hire workers and translates into higher unemployment in the economy.

In contrast, entrepreneurs in the 1K model lack the margin of adjusting intangible capital, and so reduce consumption instead, while using internal funds to prevent a greater decline in tangible capital. The decrease in tangible capital in the 1K model leads to a

\[ Y_t = \bar{k}_{it}^{\alpha_{Ki}}Z_t^{\alpha_{Z}}n_t^{1-\alpha_{Z} - \alpha_{Ki}} \]

where \( \bar{k}_{it} \) is the intangible to tangible capital ratio in steady-state: \( \bar{k}_{it} = \bar{K}_{it}/\bar{K}_i^{\alpha_{K}} \).

---

7 In this specification the production function of the model becomes:
smaller decline in output than in the benchmark model, which leads to a smaller decline in the marginal product of labor and thus in vacancies and employment, as the value of hiring workers remains relatively unaffected.\(^8\)

In summary, the amplification mechanisms of financial shocks in the two models differ as entrepreneurs use different margins of adjustment to prevent a greater decline in pledgeable assets. In the benchmark model entrepreneurs finance tangible assets by reducing intangible investment, which leads to big decreases in output, the marginal product of labor and employment. In the 1K model entrepreneurs reduce consumption and leave investment relatively unchanged, which results in a muted effect on output and employment. Finally, debt falls more in the 1K model than in the benchmark model, inasmuch as tangible capital falls more in the former than in the latter.

6. The role of search frictions in the labor market

In this section, we compare the baseline model with a specification with a frictionless labor market (2K No-Search). Figure (5) plots the impulse response functions to a one-standard deviation negative financial shock at a monthly frequency of selected variables in the baseline model and the alternative specification, so as to contrast the amplification and propagation mechanisms in the two models.

The 2K No-Search model differs from the baseline in that it has no search frictions; labor is endogenously supplied by households and wages are equal to the marginal product of labor and are not determined by Nash-bargaining as in the benchmark environment. This alternative specification has been calibrated to have the same steady state as the

---

\(^8\)Note that the stock of total capital falls similarly in both models, but the marginal product of labor declines more in the benchmark model. This is explained by the fact that the large decline in intangible investments in the benchmark model results in a greater fall in output given the way it enters the production function, which is different from its weight on the aggregate capital stock. The log-linearized equation of the aggregate capital stock is:

\[
k_t = \frac{k_{Iss}}{k_{Iss} + k_{Iss}} \hat{k}_{Iss} + \frac{k_{Iss}}{k_{Iss} + k_{Iss}} \hat{k}_{Iss}
\]

where the hat variables indicate percentage deviations from steady state, and \(k_{Iss}\) and \(k_{Iss}\) are the steady-state values of intangible and tangible capital, respectively. The capital stock relevant for the production function, and for the marginal product of labor, is different: (in log-deviations from steady-state):

\[
\hat{k}_t^P = \alpha_{KT} \hat{k}_{T, t} + \alpha_{KI} \hat{k}_{I, t}
\]

In the 1K model both the aggregate capital stock (\(\hat{k}_t\)) and the relevant capital stock for the production function (\(\hat{k}_t^P\)) decline in a similar proportion because \(\hat{k}_{T, t} = \hat{k}_{I, t} = \hat{k}_t\) and \(\hat{k}_t^P = (\alpha_{KT} + \alpha_{KI}) \hat{k}_{T, t}\). In this specification, entrepreneurs reduce consumption to prevent a large decline in \(\hat{k}_{T, t}\), which at the same time prevents a large decline in the marginal product of labor. In the benchmark model, however, the decline in \(\hat{k}_{I, t}\) is much bigger than the fall in \(\hat{k}_{T, t}\), which leads to a large fall in \(\hat{k}_t^P\), given the larger weight of \(\hat{k}_{I, t}\) in \(\hat{k}_t^P\). In the benchmark model the aggregate capital stock, \(\hat{k}_t\), exhibits a relatively muted response, inasmuch tangible capital has a larger weight on it because of its role as collateral.
baseline model, so differences in the impulse responses are explained by modeling differences and not steady-state elasticities. The steady-state hours worked in the model without search frictions (2K No-Search) equal the steady-state number of workers in the models with search frictions, meaning that the two models exhibit the same steady-state output, output per hours worked or output per worker, and capital-labor ratios.\(^9\)

By comparing the benchmark model with the 2K No-Search, we are able to assess the role of search frictions and how they interact with intangible capital in the transmission of financial shocks. The 2K No-Search model exhibits a similar decline in output, capital and debt as the benchmark model, thus showing the importance of intangible capital in the amplification of financial shocks. It differs from the baseline model in that the response of employment is smaller. A key difference between the two models is that without search frictions wages adjust downward alongside the fall in the marginal product of labor, giving entrepreneurs little incentives to change labor demand.

7. Conclusion

In this paper, we study the role of intangible capital in the transmission of financial shocks. We find that in an economy with two types of capital, and where only one can be used as collateral, financial shocks, unlike aggregate productivity shocks, can replicate the large fluctuations in aggregate employment and other labor market variables observed in the data. Intangible capital is key in the model for the amplification of shocks, as, when hit by an adverse shock to financing conditions, entrepreneurs use the additional flexibility of having intangible capital to better smooth consumption while maintaining their investment in tangible capital. They do this by tilting investment towards pledgeable assets while cutting intangible investments. This results in a substantial decline in intangible capital, which in turn leads to a decrease in the marginal product of labor. Given that wages do not fall one-to-one with the marginal product of labor because of Nash-Bargaining, the decline in the marginal product of labor reduces the incentive to hire workers, thus resulting in a decrease in vacancies, employment and output.

Our results suggest that understanding cyclical changes in credit and the composition of pledgeable and non-pledgeable assets is important for understanding business cycle dynamics. Given our representative firm assumption, understanding how financial shocks reallocate resources among heterogeneous producers depending on their capital composition is beyond the scope of this paper, but does seem a promising area for future research.\(^{10}\)

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\(^9\)We set the parameter governing the disutility of working in the 2K No-Search and JQ models so the number of hours worked in steady-state is equal to: \(l_{ss} = 0.95\), which is consistent with steady-state calibration of the baseline model in which \(n = 0.95\).

\(^{10}\)Recent papers by Khan and Thomas (2013) and Buera, Jaef, and Shin (2015) discuss financial shocks...
Acknowledgments


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Cahuc, P. and E. Wasmer (2001). Does intrafirm bargaining matter in the large firm’s matching model? Macroeconomic Dynamics 5(05), 742–747. 4
Falato, A., D. Kadyrzhanova, and J. Sim (2013). Rising intangible capital, shrinking debt capacity, and the us corporate savings glut. 2

in the context of heterogeneous producers but without any asset heterogeneity.


8. Tables and Figures

Table 1: Benchmark Calibration: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household’s discount factor</td>
<td>$\beta$</td>
<td>0.997</td>
</tr>
<tr>
<td>Entrepreneur’s discount factor</td>
<td>$\gamma$</td>
<td>0.980</td>
</tr>
<tr>
<td>Borrowing constraint parameter</td>
<td>$\bar{\chi}$</td>
<td>0.72</td>
</tr>
<tr>
<td>Depreciation of tangible capital</td>
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</tr>
<tr>
<td>Depreciation of intangible capital</td>
<td>$\delta_I$</td>
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<tr>
<td>Income share of tangible capital</td>
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<tr>
<td>Income share of intangible capital</td>
<td>$\alpha_{K_I}$</td>
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</tr>
<tr>
<td>Workers’ bargaining power</td>
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</tr>
<tr>
<td>Exogenous separation rate</td>
<td>$\chi$</td>
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</tr>
<tr>
<td>Matching function’s efficiency</td>
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<tr>
<td>Disutility of work parameter</td>
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<tr>
<td>Recruiting costs</td>
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Table 2: Stochastic Properties of Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tbody>
<tr>
<td>TFP shocks from equation (18)</td>
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<td></td>
</tr>
<tr>
<td>Autoregressive parameter for productivity shocks</td>
<td>$\rho_z$</td>
<td>0.9516</td>
</tr>
<tr>
<td>Standard deviation productivity shocks</td>
<td>$\sigma_z$</td>
<td>0.0033</td>
</tr>
<tr>
<td>Financial shocks from equation (19)</td>
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<tr>
<td>Autoregressive parameter for financial shocks</td>
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<td>0.9950</td>
</tr>
<tr>
<td>Standard deviation financial shocks</td>
<td>$\sigma_y$</td>
<td>0.0044</td>
</tr>
<tr>
<td>VAR both shocks from equation (22)</td>
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<td></td>
</tr>
<tr>
<td>Standard deviation productivity shocks</td>
<td>$\sigma_z$</td>
<td>0.0033</td>
</tr>
<tr>
<td>Standard deviation financial shocks</td>
<td>$\sigma_y$</td>
<td>0.0036</td>
</tr>
<tr>
<td>Covariance parameter innovations</td>
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<td>Autoregressive parameter for financial shocks</td>
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<tr>
<td>Spill-over terms</td>
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<tr>
<td>$\rho_{xz}$</td>
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</tr>
<tr>
<td>$\rho_{yz}$</td>
<td>0.1241</td>
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Table 3: **Business Cycle Statistics**

<table>
<thead>
<tr>
<th>Volatility Relative to Output</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TFP Shocks</td>
</tr>
<tr>
<td>Employment</td>
<td>0.56</td>
<td>0.18</td>
</tr>
<tr>
<td>Capital/Output</td>
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<td>0.99</td>
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<tr>
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<tr>
<td>Intangible/Tangible Capital</td>
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<td>0.02</td>
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</tbody>
</table>

**Correlations with Output**

<table>
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<tr>
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<th>Model</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>TFP Shocks</td>
</tr>
<tr>
<td>Employment</td>
<td>0.81</td>
<td>0.98</td>
</tr>
<tr>
<td>Aggregate Investment</td>
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<td>0.98</td>
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<tr>
<td>Market Tightness</td>
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</tr>
<tr>
<td>Intangible / Tangible Capital</td>
<td>0.54</td>
<td>0.46</td>
</tr>
</tbody>
</table>

**Labor-market variables cross- correlations (U.S. Data)**

<table>
<thead>
<tr>
<th></th>
<th>Vacancies</th>
<th>Market Tightness</th>
<th>Labor Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacancies</td>
<td>1.00</td>
<td>0.97</td>
<td>0.64</td>
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<tr>
<td>Market Tightness</td>
<td>-</td>
<td>1.00</td>
<td>0.61</td>
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**Labor-market variables cross- correlations (Model with TFP shocks)**

<table>
<thead>
<tr>
<th></th>
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<th>Labor Productivity</th>
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</thead>
<tbody>
<tr>
<td>Vacancies</td>
<td>1.00</td>
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<td>0.90</td>
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<tr>
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<td>-</td>
<td>1.00</td>
<td>0.97</td>
</tr>
</tbody>
</table>

**Labor-market variables cross- correlations (Model with Financial Shocks)**

<table>
<thead>
<tr>
<th></th>
<th>Vacancies</th>
<th>Market Tightness</th>
<th>Labor Productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacancies</td>
<td>1.00</td>
<td>0.93</td>
<td>0.41</td>
</tr>
<tr>
<td>Market Tightness</td>
<td>-</td>
<td>1.00</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: All variables, model and data, are quarterly. Output and investment are from NIPA, and in per-capita terms using civilian non-institutional population from the FRED database of the Federal Reserve Bank of St. Louis. Employment and unemployment rates are the ratio of the number of civilian employment, 16 years and over, divided by the Labor Force from the BLS. Vacancies are the quarterly average of the monthly composite Help-Wanted index from Barnichon (2010). Labor productivity is measured as output per worker. The market tightness is defined as the ratio of vacancies to the unemployment rate. The sample period is: 1980.Q1-2016.Q4. Series are detrended using a linear trend.
Figure 1: TFP and Financial Series

Note: The graph plots the TFP ($\hat{z}_t$) (dotted line) and Financial Shocks ($\chi_t$) (solid line) computed using equations (18) and (19), respectively.
Note: The graph shows the data (solid black lines) and the simulations of selected variables of the model with TFP shocks (dashed lines), financial shocks (dotted line), and both shocks (solid lines with circles). Simulations and the data variables are presented as percentage deviations from a linear trend, with the exception of the unemployment rate, which is plotted as absolute deviations from trend. All variables are at a quarterly frequency.
Figure 3: **Impulse Responses: TFP and Financial Shock**

Note: The graph plots the impulse response function of the main variables of the benchmark model to a one-standard deviation TFP ($z_t$) (dotted lines), and financial shocks ($\chi_t$) (solid lines), at a monthly frequency. The impulse-response functions are presented as percentage deviation from steady-state, with the exception of unemployment which is plotted as absolute deviations from steady-state.
Figure 4: Impulse Responses: Benchmark and 1K Model

Note: This graph plots the impulse response function of the main variables of the benchmark (solid lines) and the one capital model (1K)(dotted lines) to a one-standard deviation financial shock ($\chi_t$) at a monthly frequency. The impulse-response functions are presented as percentage deviation from steady-state, with the exception of unemployment which is plotted as absolute deviations from steady-state.
Figure 5: Impulse Responses: Benchmark and Alternative Specifications

Note: This graph plots the impulse response function of the main variables of the benchmark model (solid lines) and an alternative specification with a frictionless labor market (2K No Search -dotted lines) to a one-standard deviation financial shock ($\chi_t$) at a monthly frequency. The impulse-response functions are presented as percentage deviation from steady-state.
9. Appendix

9.1. Wage Determination

We present here the characterization of the wage using the Nash bargaining solution. The bargained wage maximizes the following Nash product

\[ W = \arg\max \left( \tilde{V}_n^\phi \left( \tilde{J}_n \right)^{1-\phi} \right) \]  

(23)

where \( \tilde{V}_n \) is the marginal value for a household of having a worker employed at wage \( W \) and \( \tilde{J}_n \) is the marginal value of a worker for the firm. The workers’ bargaining power is represented by \( \phi \). Taking logs to the Nash product and deriving with respect to the equilibrium wage \( \bar{W} \), we get the following expression:

\[ \frac{\phi \tilde{V}'_n(\bar{W})}{\tilde{V}_n(\bar{W})} + (1 - \phi) \frac{\tilde{J}'_n(\bar{W})}{\tilde{J}_n(\bar{W})} = 0 \]  

(24)

Using a recursive representation for the household’s problem, we can compute the marginal value of an employed member at the equilibrium wage:

\[ V_n(D, n) = \bar{W} - \frac{\bar{W}}{c} + \beta(1 - x - f(\bar{\theta}))EV_n(D', n'), \]  

(25)

where \( V(D, n) \) is the household’s value function. From this expression, we can compute the marginal value of having a worker employed at an arbitrary wage \( W \) for one period, and the equilibrium wage thereafter:

\[ \tilde{V}_n(W) = \frac{W - \bar{W}}{c} + V_n(\bar{D}, \bar{n}), \]  

(26)

where the first term is the incremental income from receiving the arbitrary wage rather than the equilibrium wage \( W - \bar{W} \), measured in terms of marginal utility. The second term is the marginal value of having a worker employed at the equilibrium wage.

We next turn to firms. Expressing their problem recursively, we obtain the following first order condition for the number of vacancies:

\[ q = \omega(\theta) \gamma J_n(n'). \]  

(27)

where \( J \) is the firm’s value function. The envelope condition for employment is:

\[ J_n(n) = MPL - qv - \bar{W} + (1 - x + \nu \omega(\theta))\gamma J_n(n'). \]  

(28)
Using this expression, we can rewrite the first order condition with respect to the number of vacancies as:

\[ EJ_n(n') = \frac{q}{\omega(\theta) \gamma} \]  

(29)

and plug it in the envelope condition to eliminate the continuation value:

\[ J_n(n) = MPL - qv - \bar{W} + (1 - x + \nu\omega(\theta)\gamma - \frac{q}{\omega(\theta) \gamma}). \]  

(30)

Simplifying, we obtain:

\[ J_n(n) = MPL + \frac{1 - x}{\omega(\theta)} q - \bar{W} \]  

(31)

Finally, the marginal profit of employing a worker at an arbitrary wage \( W \) for a firm and at the equilibrium wage \( \bar{W} \) thereafter is given by:

\[ \tilde{J}_n(W) = (\bar{W} - W) + J_n(\bar{n}). \]  

(32)

We can plug in the expressions for \( \tilde{V}_n(\bar{W}), \tilde{V}'_n(\bar{W}), \tilde{J}_n(\bar{W}) \) and \( \tilde{J}'_n(\bar{W}) \), to obtain the following expression:

\[ \phi \frac{1}{\tilde{v}_n(D, \bar{n})} = \frac{(1 - \phi)}{J_n(\bar{n})} \]  

(33)

Replacing \( V_n(D, \bar{n}) \), we get:

\[ \phi J_n(\bar{n}) = (1 - \phi)\tilde{c} \left[ \frac{\bar{W}}{\tilde{c}} - \varphi + \beta(1 - x - f(\bar{\theta}))E \phi J_n(n') \right] \]  

(34)

which after replacing \( V_n(D', n') \) becomes:

\[ \phi J_n(\bar{n}) = (1 - \phi)\tilde{c} \left[ \frac{\bar{W}}{\tilde{c}} - \varphi + \beta(1 - x - f(\bar{\theta}))E \frac{\phi J_n(n')}{(1 - \phi)\tilde{c}} \right]. \]  

(35)

Simplifying this expression we get:

\[ \phi J_n(\bar{n}) = (1 - \phi)\bar{W} - (1 - \phi)\varphi + \beta(1 - x - f(\bar{\theta}))E \phi J_n(n'). \]  

(36)

Using the first order condition for the number of vacancies and the envelope condition for employment, we replace \( J_n(\bar{n}) \) and \( EJ_n(n') \):

\[ \phi \left( MPL + \frac{1 - x}{\omega(\theta)} q - \bar{W} \right) = (1 - \phi)\bar{W} - (1 - \phi)\tilde{c}\varphi + \beta\phi(1 - x - f(\bar{\theta}))\frac{q}{\omega(\theta) \gamma}. \]  

(37)
Simplifying, we get the final expression for the wage condition:

\[
\bar{W} = \phi \left( MPL + \tilde{\theta}q^\beta + \frac{1 - x}{\omega(\theta)} q \left( 1 - \frac{\beta}{\gamma} \right) \right) + (1 - \phi) \varphi \bar{c}
\] (38)

9.2. Estimation of Stochastic Processes

We use quarterly data to construct our series of TFP and the financial shock, but our model is specified for a monthly frequency. Hence, we need to transform our quarterly estimates into monthly numbers. In order to do so, we solve the non-linear system of equations associated to the problem of temporal aggregation as follows:\(^{11}\)

The monthly auto-regressive process can be represented as: \(\hat{X}_t^M = A \hat{X}_{t-1}^M + B \xi_t^M\), where

\[
\hat{X}_t^M = \begin{bmatrix} z_t^M \\ \hat{z}_t \end{bmatrix}, \quad A = \begin{bmatrix} \rho_{z_{z}}^M & \rho_{z_{\chi}}^M \\ \rho_{\chi_{z}}^M & \rho_{\chi_{\chi}}^M \end{bmatrix}, \quad B = \begin{bmatrix} \sigma_{z_{z}}^M & \sigma_{z_{\chi}}^M \\ \sigma_{\chi_{z}}^M & \sigma_{\chi_{\chi}}^M \end{bmatrix}
\]

and \(\xi_t^M \sim N(0, \sum_{z,\chi} = I_2)\). We can re-write it as \(\hat{X}_t^M = A^3 \hat{X}_{t-3}^M + B \xi_{t-1}^M + B^2 \xi_{t-2}^M\), and then, re-stated in quarterly terms \(\hat{X}_t^Q = C \hat{X}_{t-1}^Q + D \xi_t\), where \(\hat{X}_t^Q = \begin{bmatrix} z_t^Q \\ \hat{z}_t \end{bmatrix}, \quad C = \begin{bmatrix} \rho_{z_{z}}^Q & \rho_{z_{\chi}}^Q \\ \rho_{\chi_{z}}^Q & \rho_{\chi_{\chi}}^Q \end{bmatrix}, \quad B = \begin{bmatrix} \sigma_{z_{z}}^Q & \sigma_{z_{\chi}}^Q \\ \sigma_{\chi_{z}}^Q & \sigma_{\chi_{\chi}}^Q \end{bmatrix}\),

\(A^3 \hat{X}_{t-3}^M = C \hat{X}_{t-1}^Q\) and \(D \xi_t = B \xi_{t-1}^M + AB \xi_{t-1}^M + A^2 B \xi_{t-2}^M\), with \(\xi_t \sim N\left(0, \sum_{z,\chi} = I_2\right)\). From our quarterly estimates we get matrices \(C\) and \(D\). The matrix \(A\), governing the persistence of the monthly process can be calculated using \(A = C^3\). We can recover the B matrix, by solving the following non-linear system: \(DD' = BB' + ABB'A' + A^2 BB'A'^2\).

\(^{11}\)For a discussion on this topic see Marcellino (1999)